USE OF DISJUNCTIVE CONSTRAINTS TO REPRESENT RISK AVERSION POLICIES

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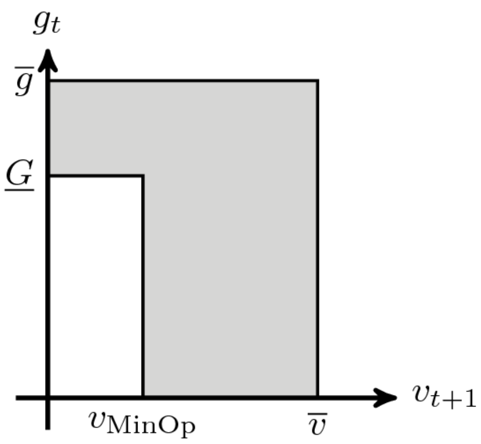
Outline

Modeling non-convex sets

Modeling non-convex sets using binary variables

Is there any simple way of modeling the following set?



Modeling non-convex sets using binary variables

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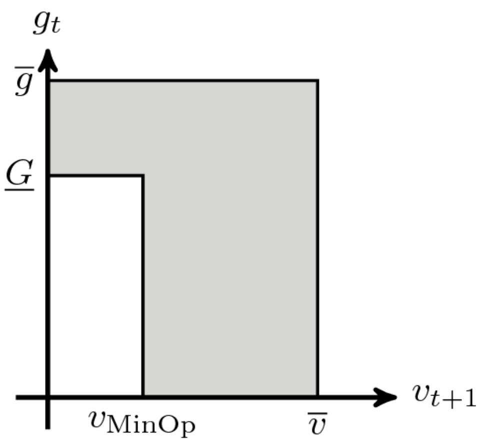
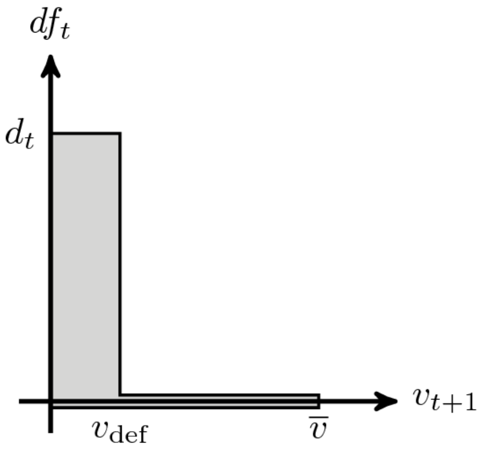


Mathematical formulation:

$$
\begin{aligned}
(1 - z) \cdot v_{\mathrm{m}} \leq v \leq \overline{v}, \\
z \cdot \underline{g} \leq g \leq \overline{g}, \\
z \in \{0, 1\}.
\end{aligned}
$$

Modeling non-convex sets using binary variables

What about this one?



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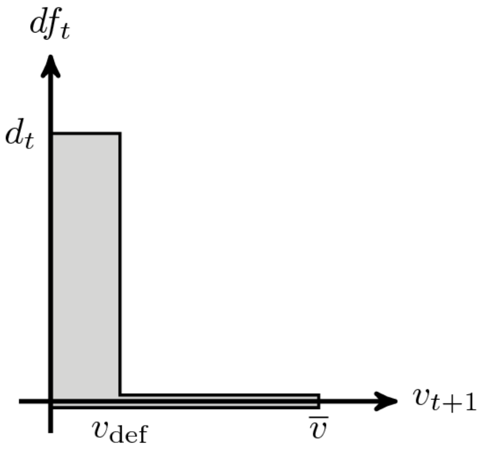

Mathematical formulation:

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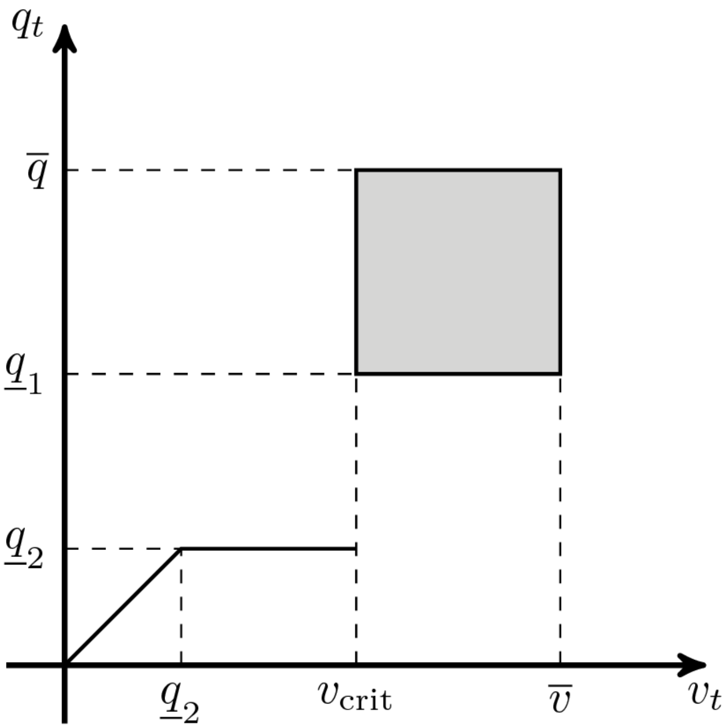
\begin{aligned}
0 \leq\ & v \leq (1 - z) \cdot \overline{v} + z \cdot v_{\mathsf{def}}, \\
0 \leq\ & df \leq z \cdot d, \\
z \in\ & \{0, 1\}.
\end{aligned}
$$

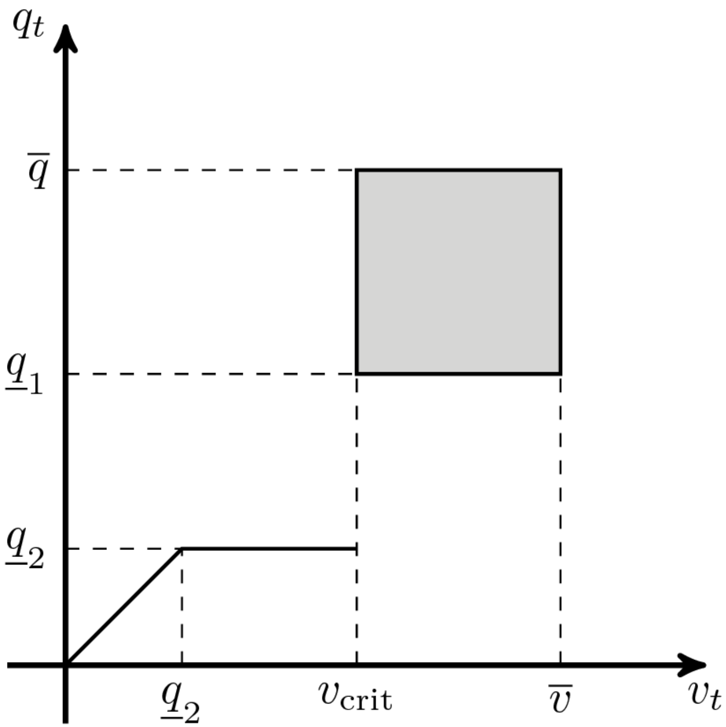
Modeling non-convex sets using binary variables

How can we formulate this set?



Modeling non-convex sets using binary variables

How can we formulate this set? Now it seems harder.



In the general case, our feasible sets are like Tangrans:

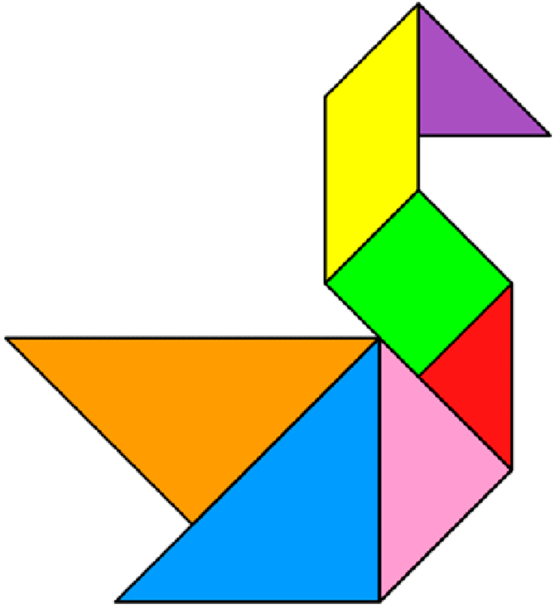


Figure: Tangran feasible set.

Disjunctive Constraints

Union of polyhedra

Let $P_i = \{x \in \mathbb{R}^n \mid A_i x \leq b_i\}$, for $i \in I$. How can we represent the corresponding union $\bigcup_{i \in I} P_i$?

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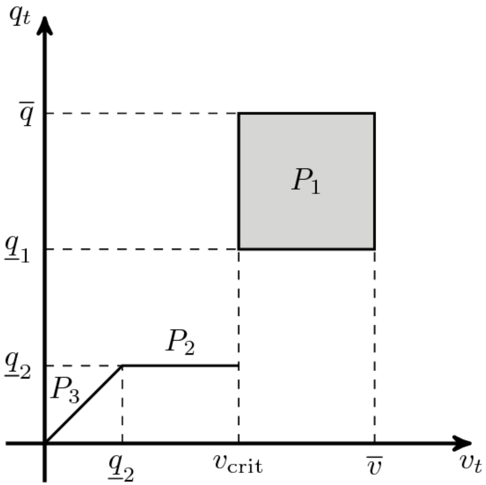
Balas's formula [Balas, 1979],[Balas, 1998]

The "magic" formula:

$$Q = \left\{ x \in \mathbb{R}^n \;\middle|\; \begin{array}{c} A_i x_i \leq z_i \cdot b_i, \\ \sum_{i \in I} x_i = x,\ \sum_{i \in I} z_i = 1, \\ x_i \in \mathbb{R}^n,\ z_i \in \{0,1\},\ i \in I. \end{array} \right\}$$

Balas's formula example

Consider the following polyhedra:



$$P_1 = \{(v,q) \mid \underline{q}_1 \le q \le \overline{q},\ v_{\text{crit}} \le v \le \overline{v}\}$$
$$P_2 = \{(v,q) \mid q = \underline{q}_2,\ \underline{q}_2 \le v \le v_{\text{crit}}\}$$
$$P_3 = \{(v,q) \mid q = v,\ 0 \le q \le \underline{q}_2\}$$

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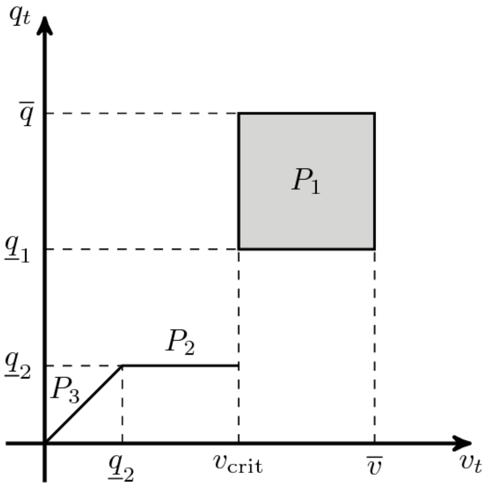

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Then, by Balas's formula

$$\bigcup_{i=1}^{3} P_i = \left\{ (q, v) \;\middle|\; \qquad \right\}.$$

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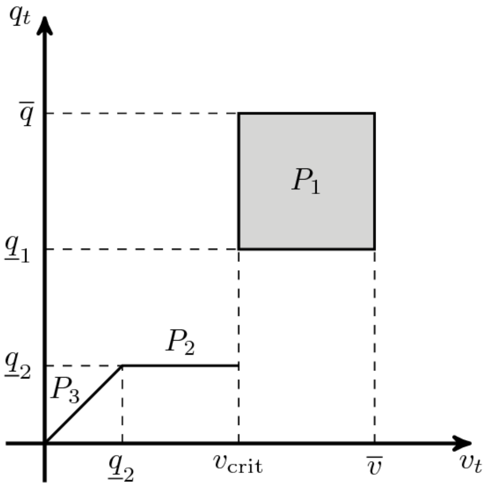

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$$\bigcup_{i=1}^{3} P_i = \left\{ (q, v) \,\middle|\, \begin{array}{ll} q = q^1 + q^2 + q^3, & z_1 + z_2 + z_3 = 1, \\ v = v^1 + v^2 + v^3, & z_i \in \{0, 1\},\ i = 1, 2, 3, \end{array} \right\}.$$

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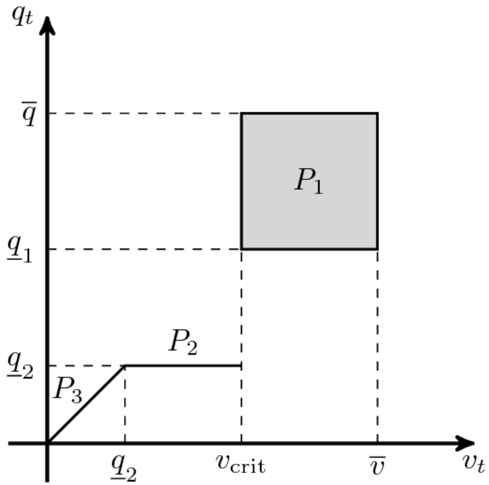

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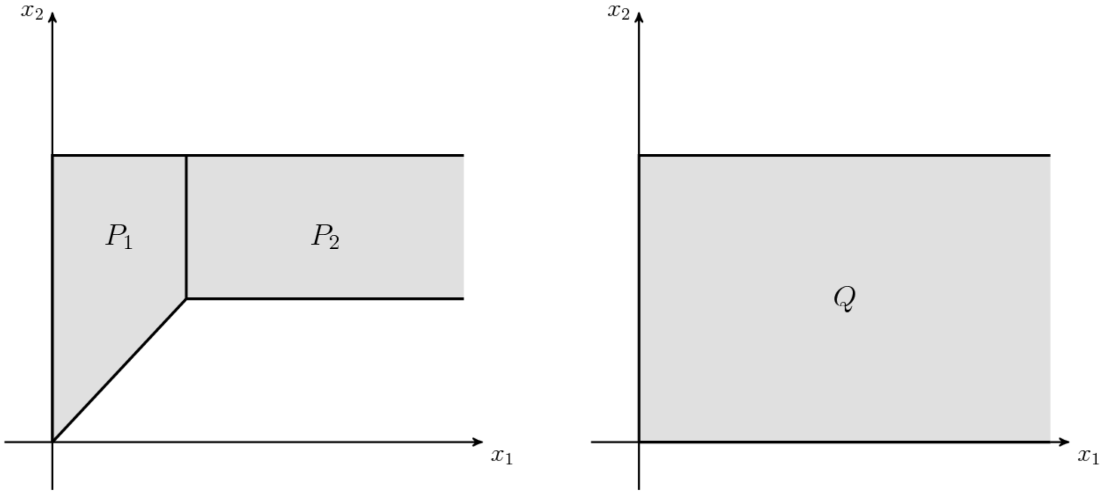
z_1 v_{\text{crit}} \le v^1 \le z_1 \overline{v}, & z_2 \underline{q}_2 \le v^2 \le z_2 v_{\text{crit}}, & 0 \le v^3 \le z_3 \underline{q}_2, \\
z_1 \underline{q}_1 \le q^1 \le z_1 \overline{q}, & q^2 = z_2 \underline{q}_2, & q^3 - v^3 = 0.
\end{array} \right\}.
$$

When Balas's formula fails

Theorem (Jeroslow, [Jeroslow, 1987])

If each P_i is non-empty and Balas's formula does not represent $\bigcup_{i=1}^{p} P_i$, then no set of linear constraints involving continuous and binary variables can do it.



(a) Non-representable set. (b) Certificate of non-representability.

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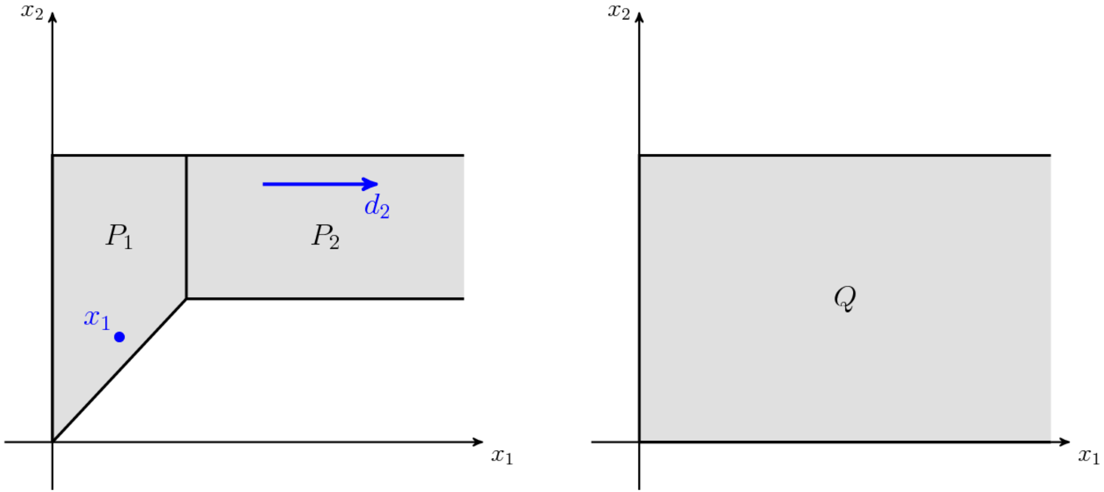

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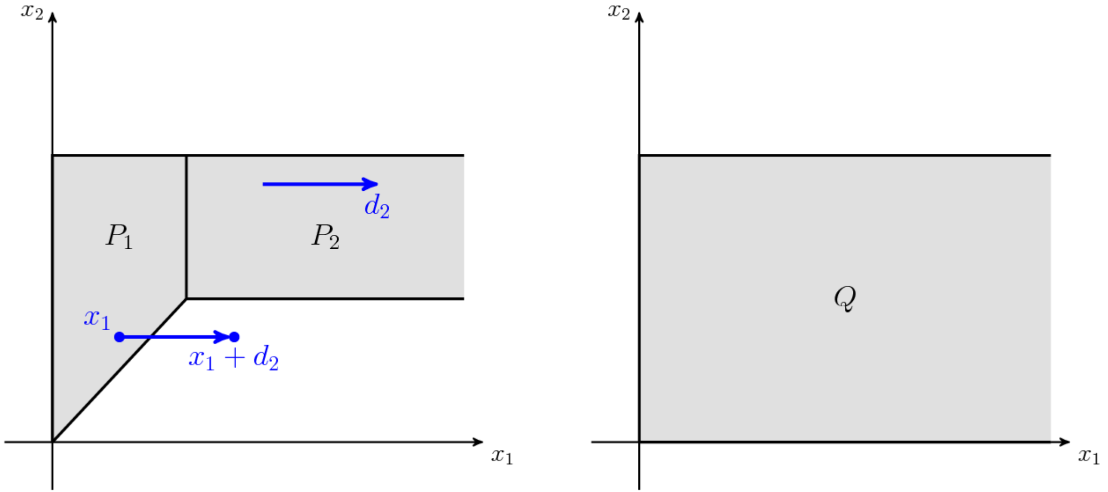

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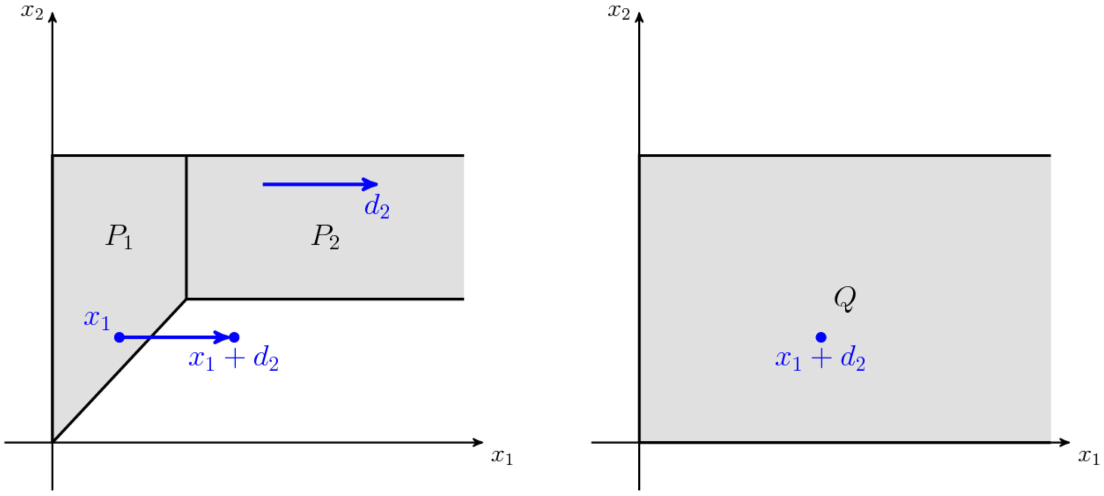

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Case study: Low storage risk aversion

Long term hydrothermal operation planning model

$$
\begin{aligned}
Q_t(v_t, \boldsymbol{a}_t) = \quad \min \quad & c^\top g_t + c_{df}^\top df_t + \beta \overline{Q}_{t+1}(v_{t+1}) \\
\text{s.t.} \quad & v_{t+1} = v_t + \boldsymbol{a}_t - q_t - s_t \\
& q_t + M_I g_t + M_D f_t + df_t = d_t \\
& (v_{t+1}, q_t, s_t, g_t, df_t, f_t) \in \mathcal{X}_t
\end{aligned}
$$

$$
\overline{Q}_{t+1}(v_{t+1}) = \begin{cases} \mathbb{E}\left[Q_{t+1}(v_{t+1}, \boldsymbol{a}_{t+1})\right] & , t \in \{1, \ldots, T-1\} \\ 0 & , t = T \end{cases}
$$

Variables:

- v_t: Stored energy at the beginning of stage t
- q_t: Hydro generation during stage t
- s_t: Spilled energy during stage t
- g_t: Thermal generation during stage t
- df_t: Deficit (load shedding) during stage t
- f_t: Energy interchange between subsystems during stage t

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$$\overline{Q}_{t+1}(v_{t+1}) = \begin{cases} \sum_{i=1}^{N_t} p_t^i \cdot Q_{t+1}(v_{t+1}, \boldsymbol{a}_t^i) & , t \in \{1, \dots, T-1\} \\ 0 & , t = T \end{cases}$$

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Operational planning model

- System configuration taken from January 2015;
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- 20% uniform level for v_{Safe}, over all months;
- 5% increase in demand to highlight the effects of the different policies;

- policy calculation with SDD(i)P, 1000 forward iterations;
- policy evaluation for the first 36 months, 2000 series.

Base case: Risk-Neutral

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{Q}_{t+1}(v_{t+1})$$

$$\text{s.t.} \quad (*)_{\text{P}}.$$

$$\overline{Q}_{t+1}(v_{t+1}) = \begin{cases} \mathbb{E}\left[Q_{t+1}(v_{t+1}, a_{t+1})\right] & , \quad t \in \{1, \dots, T-1\}, \\ 0 & , \quad t = T, \end{cases}$$

Penalty approach: add a term to the objective function

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{\mathcal{Q}}_{t+1}(v_{t+1}) + \theta_t^\top (v_{\text{Safe}} - v_{t+1})_+$$

$$\text{s.t.} \quad (*)_{\text{P}}.$$

$$\overline{\mathcal{Q}}_{t+1}(v_{t+1}) = \begin{cases} \mathbb{E}\left[Q_{t+1}(v_{t+1}, a_{t+1})\right] & , \quad t \in \{1, \ldots, T-1\}, \\ 0 & , \quad t = T, \end{cases}$$

Disjunctive Constraints representation

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{Q}_{t+1}(v_{t+1})$$

$$\begin{aligned}
\text{s.t.} \quad & (*)_{\mathrm{P}}. \\
& z_g \cdot v_{\text{Safe}} \leq v_{t+1} \leq (1 - z_d) \cdot \bar{v}, \\
& M_I g_t \geq (1 - z_g) \cdot G_{\text{safe}}, \quad 0 \leq df_t \leq z_d \cdot d_t, \\
& z_g, z_d \in \{0, 1\}^{N_{\text{sys}}}.
\end{aligned}$$

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Use of the CVaR Risk Measure

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{\mathcal{Q}}_{t+1}(v_{t+1})$$

$$\text{s.t.} \quad (*)_{\text{P}}.$$

$$\overline{\mathcal{Q}}_{t+1}(v_{t+1}) = \begin{cases} \rho_t[Q_{t+1}(v_{t+1}, a_{t+1})] & , \quad t \in \{1, \ldots, T-1\}, \\ 0 & , \quad t = T, \end{cases}$$

where $\rho_t[Z] = (1-\lambda)\mathbb{E}[Z] + \lambda \text{CVaR}_\alpha[Z]$.

CVaR with penalization

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{Q}_{t+1}(v_{t+1}) + \theta_t^\top (v_{\text{Safe}} - v_{t+1})_+$$

$$\text{s.t.} \quad (*)_{\text{P}}.$$

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CVaR with Disjunctive constraints

$$Q_t(v_t, a_t) = \min_{x_t \in \mathcal{X}_t} \quad c_g^\top g_t + c_{df}^\top df_t + \beta \overline{\mathcal{Q}}_{t+1}(v_{t+1})$$

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$$z_g \cdot v_{\text{Safe}} \leq v_{t+1} \leq (1 - z_d) \cdot \bar{v},$$

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$$z_g, z_d \in \{0, 1\}^{N_{\text{sys}}}.$$

$$\overline{\mathcal{Q}}_{t+1}(v_{t+1}) = \begin{cases} \rho_t[Q_{t+1}(v_{t+1}, a_{t+1})] & , \quad t \in \{1, \ldots, T-1\}, \\ 0 & , \quad t = T, \end{cases}$$

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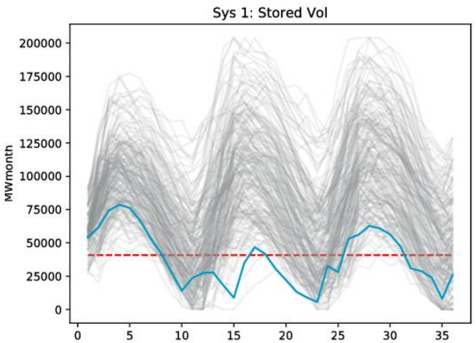
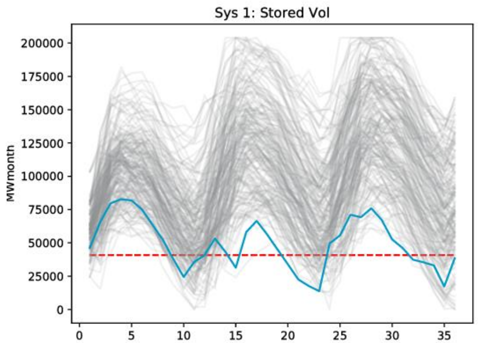
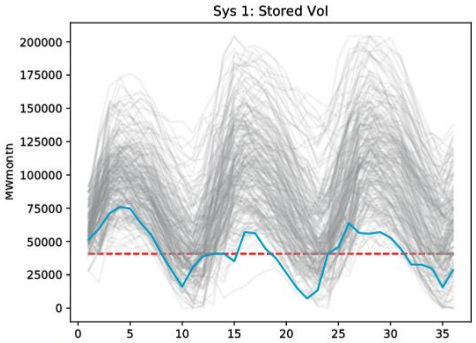
Stored energy – Southeast



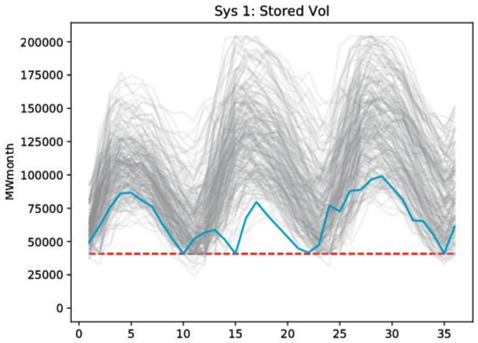
(a) Risk neutral



(b) Mean-CVaR



(c) RN + Disjunctive Constraints



(d) RN + Penalization

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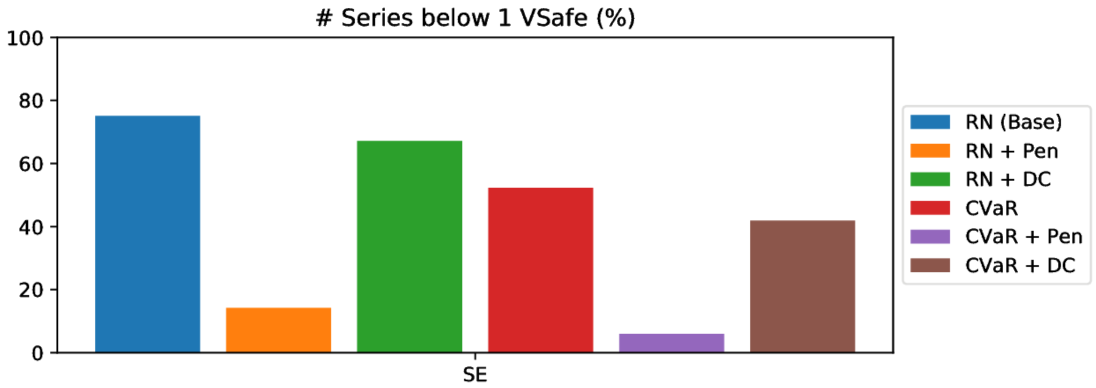
VSafe Violation and deficit – Southeast



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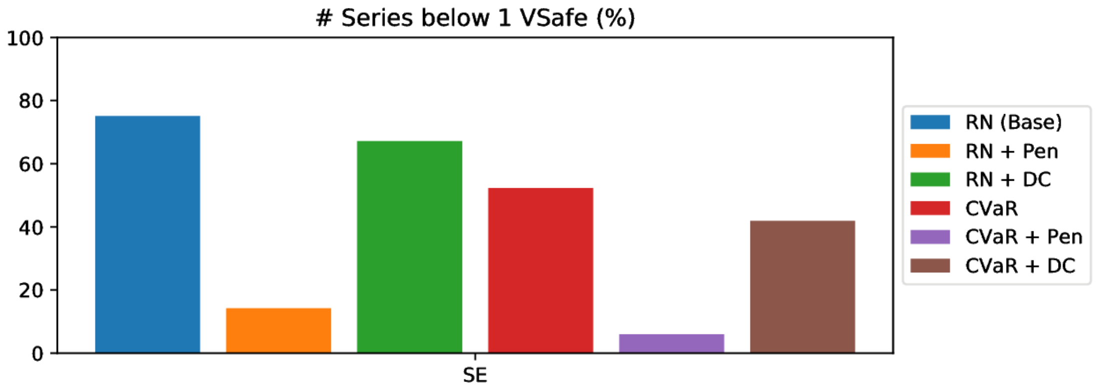

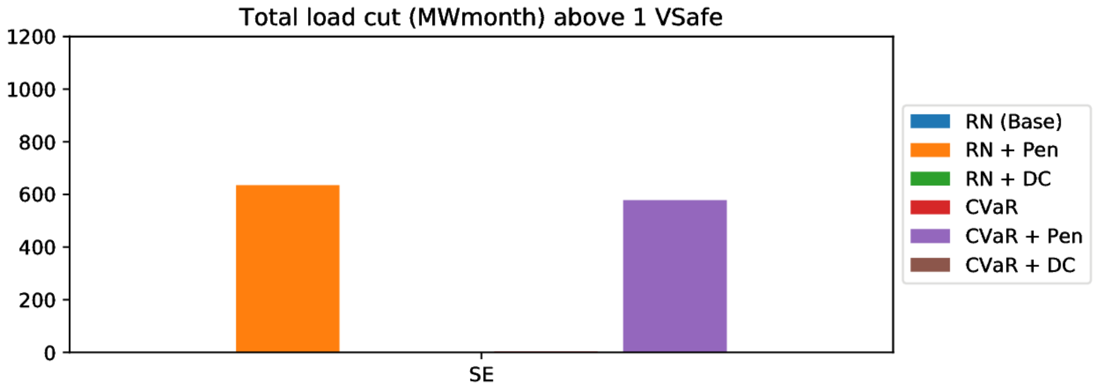

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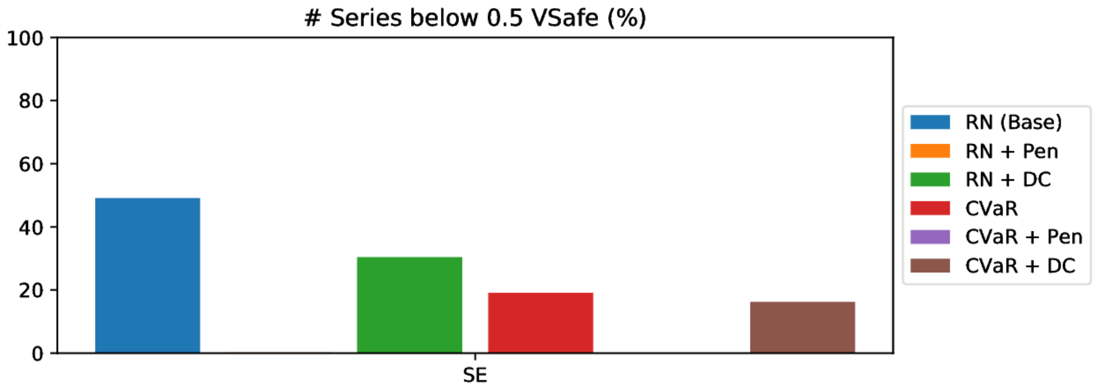

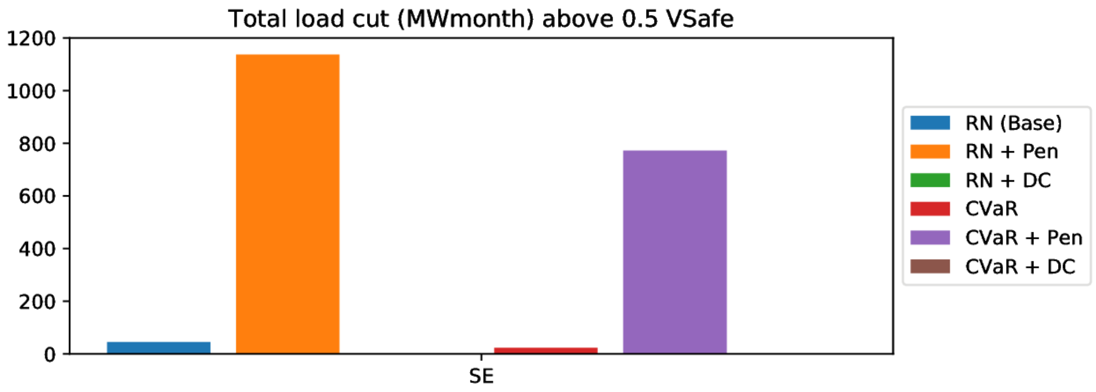

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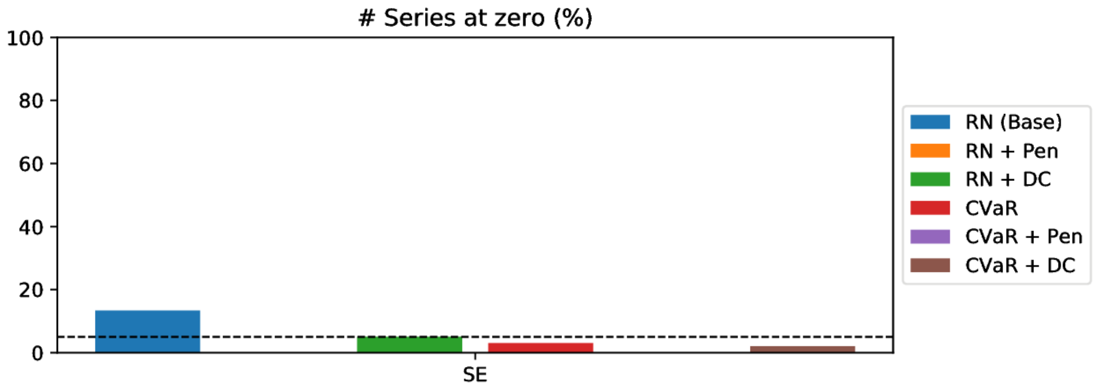
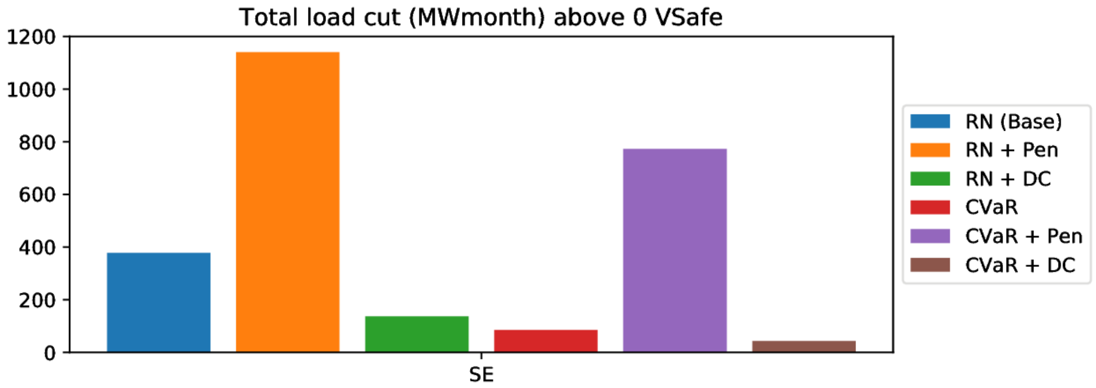
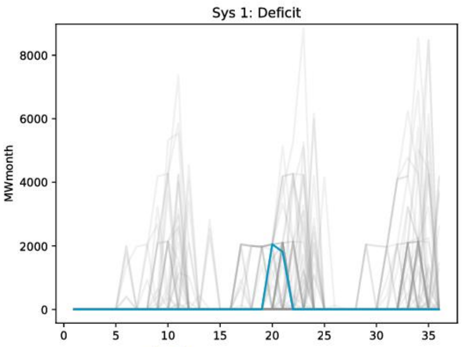




Deficit – Southeast



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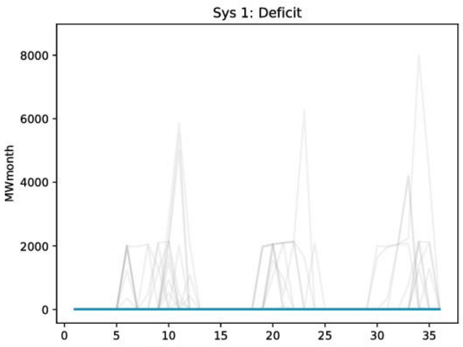

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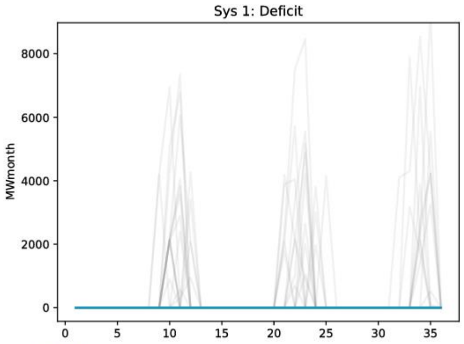

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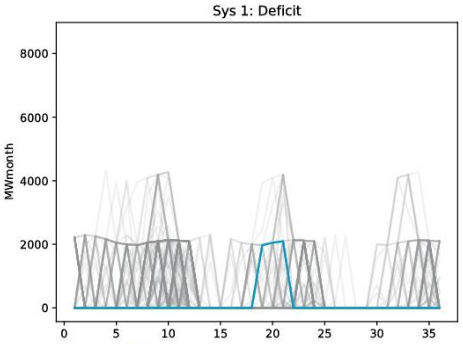

(d) RN + Penalization

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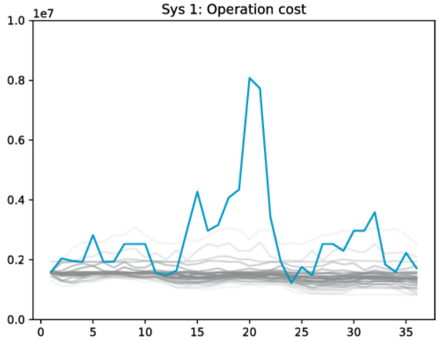
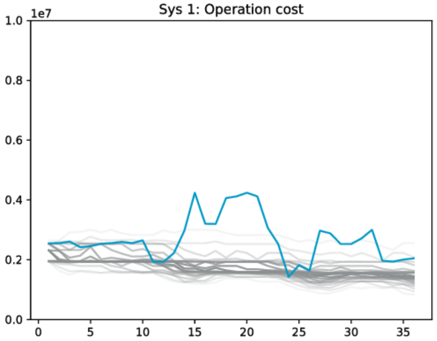
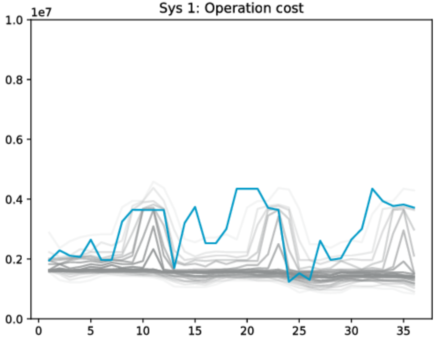
Operation cost (Quantiles)



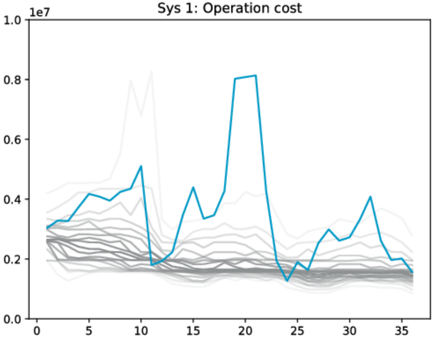
(a) Risk neutral



(b) Mean-CVaR



(c) RN + Disjunctive Constraints



(d) RN + Penalization

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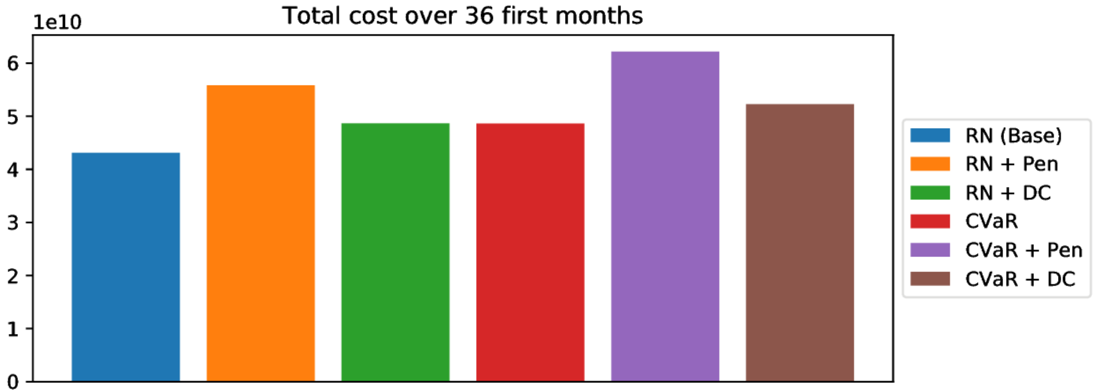
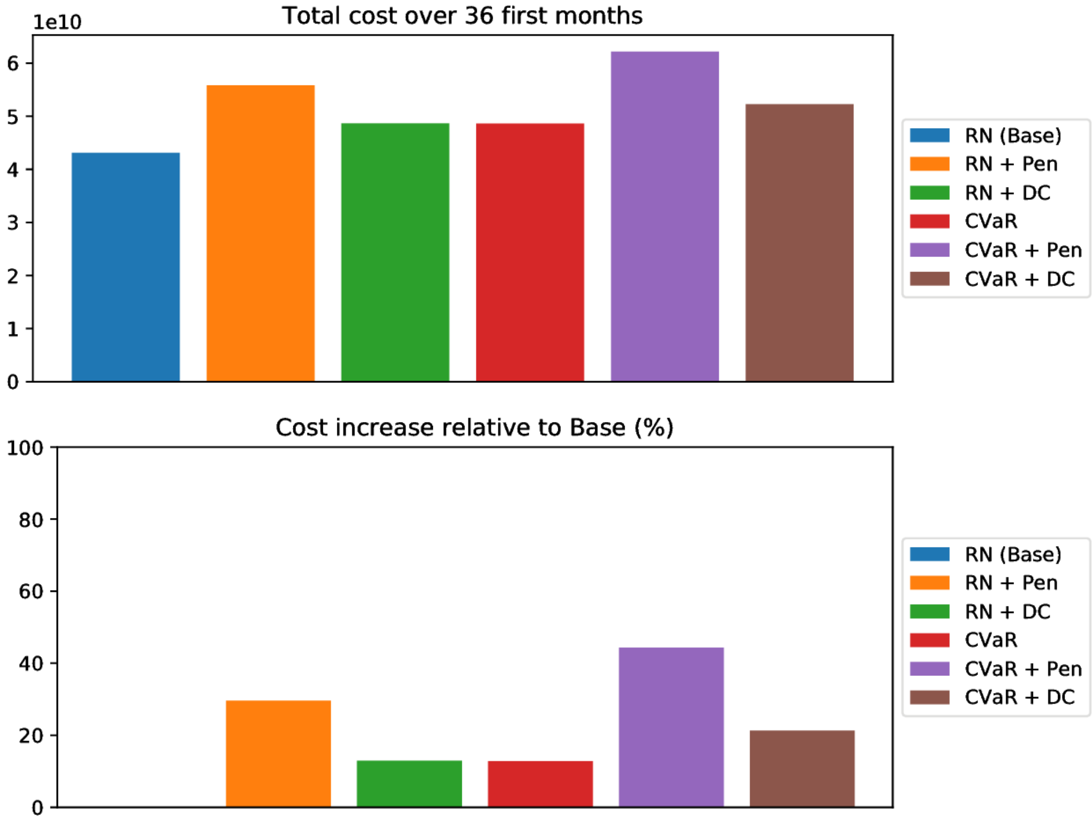
Operation cost



Operation cost



Take away

Summary

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- The penalization technique is not a good alternative since it induces high operational costs and preventive deficit;
- CVaR avoids high costs, which has an indirect impact in stored energy and thermal generation;
- The combination of CVaR and the Disjunctive Constraints is a very good alternative for low storage risk aversion;

Future work

Model extensions

Develop new representation techniques for non-convex functions of continuous variables, that do not need discretization.

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Thank You!

References I

Balas, E. (1979).
Disjunctive programming.
In *Annals of Discrete Mathematics*, pages 3–51. Elsevier.

Balas, E. (1998).
Disjunctive programming: Properties of the convex hull of feasible points.
Discrete Applied Mathematics, 89(1-3):3–44.

Jeroslow, R. G. (1987).
Representability in mixed integer programmiing, i: Characterization results.
Discrete Applied Mathematics, 17(3):223–243.

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