

**Investigation of the AV@R and minimum storage energy
target levels approach**

Final Report

**First activity of the technical cooperation between
Georgia Institute of Technology
and**

ONS - Operador Nacional do Sistema Elétrico

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1 Introduction

The Electric System National Operator (ONS) of Brazil is a private non-profitable entity created on 26 August 1998. It is responsible for coordinating and controlling the operation of generation and transmission facilities in the National Interconnected Power System (NIPS) under supervision and regulation of the Electric Energy National Agency (ANEEL).

The Brazilian power system generation is hydro dominated (about 75% of the installed capacity) and characterized by large reservoirs presenting multi-year regulation capability, arranged in complex cascades over several river basins. The hydro plants use stored water in the reservoirs to produce energy in the future, replacing fuel costs from the thermal units. Since the water inflows depend on rainfalls, the amount of future inflows is uncertain and cannot be predicted with a high accuracy. Moreover, historical records indicate possibility of some very dry time periods which, of course, can put a severe burden on hydro power generation.

Mathematical algorithms compose the core of the Energy Operation Planning Support System. The former objective was to calculate an operation strategy which minimizes the expected value of the operation costs over a planning period of time. This led to formulation of large scale multistage stochastic programming problems. *Stochastic Dual Dynamic Programming* (SDDP) method is the main algorithmic tool which currently is employed to solve the involved stochastic optimization problems.

In the previous Technical Agreement signed between Georgia Tech and ONS, that continued from September 2010 until August 2012, several topics regarding the problem formulation methodology and the SDDP algorithm were investigated and certain developments were suggested. One of the main contributions of that project was the development of a risk averse approach based on the *Average Value-at-Risk* (AV@R) (also called *Conditional Value-at-Risk*) risk measure that complies with the SDDP methodology and results in a policy that is less sensitive to extreme scenarios with a reasonable increase on average costs when compared with the risk neutral (traditional) approach. This methodology is currently in use in the planning procedures of the NIPS. The goal of the current Technical Agreement report is to continue investigation of the involved modeling and computational issues.

This document aims to report the investigations carried out from September to November 2014 regarding the use of minimum storage energy target levels approach as a means of increasing the security guidelines of the operation.

1.1 Report structure

This report is organized as follows. In the Introduction we gave a brief description of the hydrothermal operation planning problem with emphasis on the Brazilian power system case. In the next section, general methodological aspects of the mathematical formulation and modeling approach used to solve this problem is presented, with focus in the Stochastic Dual Dynamic Programming (SDDP) algorithm.

The main focus is to investigate the use of a minimum stored energy target curve for the equivalent energy reservoir, in order to limit the depletion of the reservoir, which poses operation difficulties. It is known that the use of the AV@R criterion limits the frequency of high operation costs. On the other hand, the AV@R criterion poses no restriction on completely emptying the reservoirs, a situation that results in operational difficulties. In this way, aiming at complementing the AV@R criterion, it is evaluated the combined use of both criteria.

The performance of the proposed approach to limit the minimum stored energy in the reservoirs is illustrated with the aid of some numerical experiments.

2 Objectives and technical details

Basic modeling equations for the aggregate representation of the NIPS can be described as follows. The energy conservation equation for each equivalent energy reservoir $n \in \{1, \dots, r\}$:

$$SE_{t,n} = SE_{t-1,n} + CE_{t,n} - GH_{t,n} - SP_{t,n}. \quad (1)$$

That is, the stored energy (SE) at the end of each stage (start of the next stage) is equal to the initial stored energy plus controllable energy inflow (CE) minus total hydro generated energy (GH) and losses (SP) due to spillage, evaporation, etc. At each stage, the net subsystem load L , given by the remaining load after discounting the uncontrolled energy inflow from the total load, has to be met by the total hydro, the sum of all thermal generation (GT) belonging to system n , given by the set NT_n , and the net interconnection energy flow (NF) to each subsystem.

The energy balance equation for system n is

$$GH_{t,n} + \sum_{j \in NT_n} GT_{t,j} + NF_{t,n} = L_{t,n}. \quad (2)$$

The cost of reservoir n at stage t is

$$\sum_{j \in NT_n} CT_j GT_{t,j}, \quad (3)$$

where CT_j are the unit operation cost of each thermal plant and penalty for failure in load supply. The hydroelectric generation costs are assumed to be zero. In the terminology of stochastic optimal control (in discrete time) we have here that $SE_{t,n}$ are state variables, $GH_{t,n}, GT_{t,j}, NF_{t,n}$ are control variables and $CE_{t,n}$ are disturbances (random data), $n \in \{1, \dots, r\}$.

In the risk neutral setting the objective is to minimize the expected value of the total cost. That is, the optimization problem can be written as the following stochastic programming problem

$$\text{Min}_{\substack{A_1 x_1 = b_1 \\ x_1 \geq 0}} c_1^\top x_1 + \mathbb{E} \left[\min_{\substack{B_2 x_1 + A_2 x_2 = b_2 \\ x_2 \geq 0}} c_2^\top x_2 + \mathbb{E} \left[\dots + \mathbb{E} \left[\min_{\substack{B_T x_{T-1} + A_T x_T = b_T \\ x_T \geq 0}} c_T^\top x_T \right] \right] \right]. \quad (4)$$

Constraints $B_t x_{t-1} + A_t x_t = b_t$ are obtained writing

$$x_t = (SE, GH, GT, SP, NF)_t^\top, \quad b_t = (CE, L)_t^\top, \quad c_t = (0, 0, CT, 0, 0)_t^\top,$$

$$A_t = \begin{pmatrix} I & I & 0 & I & 0 \\ 0 & I & \Delta & 0 & I \end{pmatrix}, \quad B_t = \begin{pmatrix} -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where $\Delta = \{\delta_{n,j} = 1 \text{ for all } j \in NT_n \text{ and zero else}\}$, I and 0 are identity and null matrices, respectively, of appropriate dimensions and the components of CT are the unit operation cost of each thermal plant and penalty for failure in load supply. Note that hydroelectric generation costs are assumed to be zero. Physical constraints on variables like limits on the capacity of the equivalent reservoir, hydro and thermal generation, transmission capacity and so on are taken into account with constraints on x_t . Note that only CE_t components of b_t are random here.

2.1 Minimum stored energy target approach

The goal is to control levels of the stored energy SE . Of course, one possible approach is to impose hard constraints $SE_{t,n} \geq \ell_{t,n}$, for specified low levels $\ell_{t,n}$, at every (or some specified) stage of the process. This means that every time the stored energy goes below the specified levels, expensive thermal generation units should be employed. That is, to impose limits on the minimum stored energy level can be interpreted as imposing limits to the actual energy storage capacity of the reservoirs. This is an inefficient procedure and should not be implemented.

An alternative is to try to control the stored energy levels in a dynamic fashion. There are several possible ways to approach this problem. One possible approach is to add penalty to the total cost over the considered T stages. That is, in the framework of the risk neutral formulation the expected value of the cost is supplemented by a penalty at every stage of the process, i.e., for chosen constants $\kappa_{t,n} > 0$, $n = 1, \dots, r$, the following objective is minimized¹

$$\mathbb{E} \left\{ \sum_{t=1}^T \sum_{n=1}^r \left(\sum_{j \in NT_n} CT_j GT_{t,j} + \kappa_{t,n} [\ell_{t,n} - SE_{t,n}]_+ \right) \right\}. \quad (5)$$

The objective is penalized by the difference $\ell_{t,n} - SE_{t,n}$ between the desired levels $\ell_{t,n}$ and actual levels $SE_{t,n}$ in case $SE_{t,n}$ is *less* than $\ell_{t,n}$. In this formulation the penalization is only in the objective (not in the constraints) and is *on average*. Because of that this can be viewed as a “look ahead” approach. It could be noted that by setting $\kappa_{t,n} = 0$, for some values of t and n , we remove the corresponding penalty term from the objective function. This gives a flexibility of controlling the stored energy at *specified* time stages and energy reservoirs.

Moreover, in the nested formulation the expectation in formula (5) can be replaced by appropriate risk measures, so that the energy conservation and the current risk averse approaches can be combined together. That is, in the nested form the objective becomes

$$Z_1 + \rho_{2|\xi_1} \left[Z_2 + \dots + \rho_{T|\xi_{[T-1]}}(Z_T) \right], \quad (6)$$

where $Z_t = \sum_{j \in NT_n} (C_j GT_{t,j} + \kappa_{t,n} [\ell_{t,n} - SE_{t,n}]_+)$ and $\rho_{t|\xi_{[t-1]}}$ are chosen risk measures.

In particular, if we consider

$$\rho_{t|\xi_{[t-1]}}(Z_t) = (1 - \lambda_t) \mathbb{E}[Z_t | \xi_{[t-1]}] + \lambda_t \mathbf{AV@R}_{\alpha_t}[Z_t | \xi_{[t-1]}]$$

we are in the same framework of the mean-AV@R SDDP approach nowadays in use.

Let us start with the risk neutral case (5). We assume that the data process b_1, \dots, b_T is *stagewise independent*. Recall that case of the autoregressive process can be reduced to the stagewise independent case by increasing the number of state variables. The dynamic programming equations for the (risk neutral) problem (4) can be written as

$$Q_t(x_{t-1}, b_t) = \inf_{x_t \geq 0} \left\{ c_t^\top x_t + \mathcal{Q}_{t+1}(x_t) : B_t x_{t-1} + A_t x_t = b_t \right\}, \quad (7)$$

where

$$\mathcal{Q}_{t+1}(x_t) = \mathbb{E}[Q_{t+1}(x_t, b_{t+1})], \quad (8)$$

$t = 2, \dots, T$ and $\mathcal{Q}_{T+1}(\cdot) \equiv 0$.

¹By $[a]_+ = \max\{a, 0\}$ we denote the positive part of a number a . That is, $[a]_+ = a$ if $a \geq 0$ and $[a]_+ = 0$ if $a < 0$.

In formulation (5) apart from the costs $c_t^\top x_t = \sum_{j \in NT_n} CT_j GT_{t,j}$ we have the additional terms $\sum_{j \in NT_n} \kappa_{t,n} [\ell_{t,n} - SE_{t,n}]_+$. That is, equations (7) should be adjusted to

$$Q_t(x_{t-1}, b_t) = \inf_{x_t \geq 0} \left\{ c_t^\top x_t + \sum_{n=1}^r \kappa_{t,n} [\ell_{t,n} - SE_{t,n}]_+ + Q_{t+1}(x_t) : B_t x_{t-1} + A_t x_t = b_t \right\}. \quad (9)$$

Note that the additional terms are convex with respect to the variables $SE_{t,n}$ and hence the cost-to-go functions are convex. Equivalently the cost-to-go function $Q_t(x_{t-1}, b_t)$ is given by the optimal value of the problem

$$\begin{aligned} \text{Min}_{x_t \geq 0, z_t \geq 0} \quad & c_t^\top x_t + \sum_{n=1}^r \kappa_{t,n} z_{t,n} + Q_{t+1}(x_t) \\ \text{s.t.} \quad & B_t x_{t-1} + A_t x_t = b_t, \\ & \ell_{t,n} - SE_{t,n} \leq z_{t,n}, \quad n = 1, \dots, r. \end{aligned} \quad (10)$$

We obtain a linear multistage problem to which the SDDP method can be applied in a more or less straightforward way. Note that variables $z_{t,n}$ can be considered as additional control variables, so the number of state variables (variables SE) did not change. The risk averse case can also be treated in a similar way by adding the penalty term $\sum_{n=1}^r \kappa_{t,n} [\ell_{t,n} - SE_{t,n}]_+$.

Remark 1 It is also possible to make the penalty weights $\kappa_{t,n}$ dependent on the levels $\ell_{t,n}$. That is, for levels $\ell_{t,n}^1 > \dots > \ell_{t,n}^m \geq 0$ choose weights $\delta_{t,n}^i > 0$, $i = 1, \dots, m$, and adjust dynamic equations (10) to

$$\begin{aligned} \text{Min}_{x_t \geq 0, z_t \geq 0} \quad & c_t^\top x_t + \sum_{n=1}^r \sum_{i=1}^m \delta_{t,n}^i z_{t,n}^i + Q_{t+1}(x_t) \\ \text{s.t.} \quad & B_t x_{t-1} + A_t x_t = b_t, \\ & \ell_{t,n}^i - SE_{t,n} \leq z_{t,n}^i, \quad n = 1, \dots, r, \quad i = 1, \dots, m. \end{aligned} \quad (11)$$

That is, for respective values of $SE_{t,n}$ the following penalties are added:

$$\ell_{t,n}^1 \geq SE_{t,n} \geq \ell_{t,n}^2 \Rightarrow \delta_{t,n}^1 (\ell_{t,n}^1 - SE_{t,n}), \quad (12)$$

$$\ell_{t,n}^2 \geq SE_{t,n} \geq \ell_{t,n}^3 \Rightarrow \delta_{t,n}^1 (\ell_{t,n}^1 - SE_{t,n}) + \delta_{t,n}^2 (\ell_{t,n}^2 - SE_{t,n}), \quad (13)$$

and so on

$$\ell_{t,n}^m \geq SE_{t,n} \Rightarrow \delta_{t,n}^1 (\ell_{t,n}^1 - SE_{t,n}) + \delta_{t,n}^2 (\ell_{t,n}^2 - SE_{t,n}) + \dots + \delta_{t,n}^m (\ell_{t,n}^m - SE_{t,n}). \quad (14)$$

In particular if $\delta_{t,n}^i = \gamma_{t,n}$, $i = 1, \dots, m$, then penalties (12)–(14) become

$$\gamma_{t,n} (\ell_{t,n}^1 - SE_{t,n}), \quad 2\gamma_{t,n} \left(\frac{\ell_{t,n}^1 + \ell_{t,n}^2}{2} - SE_{t,n} \right), \dots, \quad m\gamma_{t,n} \left(\frac{\ell_{t,n}^1 + \dots + \ell_{t,n}^m}{m} - SE_{t,n} \right),$$

respectively.

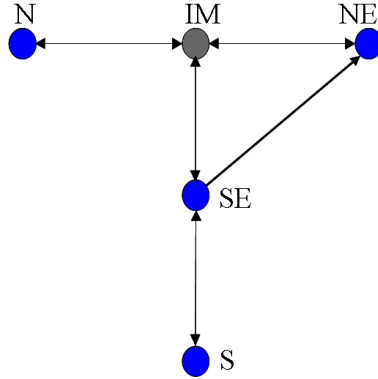


Figure 1: Aggregate representation of the Brazilian interconnected power system

	% of total load curtailment	Cost
P1	0 – 5	1206,38
P2	5 – 10	2602,56
P3	10 – 20	5439,12
P4	20 – 100	6180,26

Table 1: Deficit costs and depths

3 Case study

The numerical experiments described in this report were carried out considering instances of multi-stage linear stochastic problems based on an aggregate representation of the Brazilian Interconnected Power System long-term operation planning problem, as of January 2012. This system can be represented by a graph with four generation nodes – comprising sub-systems Southeast (SE), South (S), Northeast (NE) and North (N) – and one (Imperatriz, IM) transshipment node (see Figure 1).

The load of each area must be supplied by local hydro and thermal plants or by power flows among the interconnected areas. A slack thermal generator of high cost that increases with the amount of load curtailment accounts for load shortage at each area (Table 1). Interconnection limits between areas may differ depending on the flow direction, see Table 2. The energy balance equation for each sub-system has to be satisfied for each stage and scenario. There are bounds on stored and generated energy for each sub-system aggregate reservoir and on thermal generations.

		to				
		SE	S	NE	N	IM
from	SE	–	7700	1000	0	4000
	S	5670	–	0	0	0
	NE	600	0	–	0	3000
	N	0	0	0	–	∞
	IM	2854	0	3960	3149	–

Table 2: Interconnection limits between systems

The long-term planning horizon for the Brazilian case comprises 60 months, due to the existence of multi-year regulation capacity of some large reservoirs. In order to obtain a reasonable cost-to-go function that represents the continuity of the energy supply after these firsts 60 stages, a common practice is to add 60 more stages to the problem and consider a zero cost-to-go function at the end of the 120th stage. Hence, the objective function of the planning problem is to minimize the convex combination of the expectation and Average Value-at-Risk costs along the 120 months planning horizon, while supplying the area loads and obeying technical constraints. The total operation cost is the sum of thermal generating costs plus a penalty term that reflects energy shortage.

A scenario tree consisting of $1 \times 200 \times 100 \times 100 \times \dots \times 100$ scenarios, for 120 stages, was sampled based on a periodic autoregressive multivariate statistical model with multiplicative error for the energy inflow record. In this (seasonal) model, the empirical distribution for each month and for every system is used to represent the corresponding noise distribution. The scenario tree is generated by sampling from these empirical distributions. The input data for this statistical model is based on 80 observations of the natural energy inflow (from year 1931 to 2010) for each of the considered 4 systems.

The case's general data, such as hydro and thermal plants data and interconnections capacities were taken as static values throughout the planning horizon (120 months). The monthly seasonal demand is taken into account, but the annual value is constant. The energy inflows may vary along the stages.

All results shown in this report were obtained simulating 75 years of historical sequences with the corresponding policies.

3.1 Proof of concept

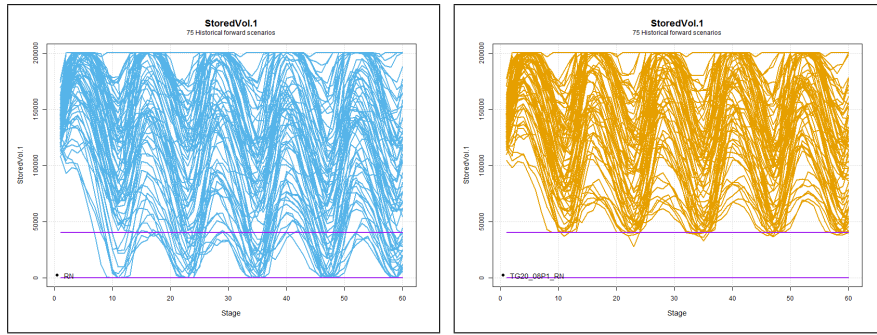
In this section, we show the effects of the imposition of the minimum stored energy target regarding the evolution of the stored volumes.

The first column of Figure 2 shows the evolution of all four subsystem with the risk neutral criterion without any additional constraint. In this case, the resulting policy uses all the available hydro energy and the reservoirs deplete completely. In the second column the minimum target level $\ell_{t,n} = 0.20$ of the maximum stored energy was enforced for all t and n with penalty value of 0.8 of the deficit cost corresponding to the load curtailment up to 5% of the load. It can be seen that the simulated stored energy volumes are above the minimum for almost all stages and subsystems.

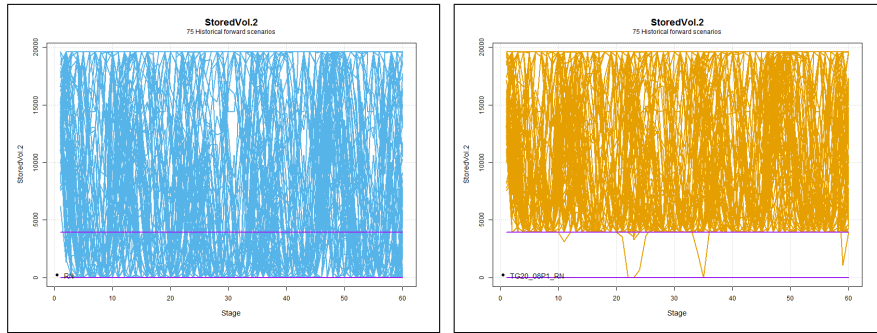
Although imposing these $\ell_{t,n}$ limits fulfils the intended purpose, the stored volumes frequently touches the limit curve and the associated operation costs tend to have high variability. On the other hand, it is known that the AV@R risk averse criterion limits the frequency of high operation costs variability but does not prevent the emptying of the reservoir. Therefore, it is instructive to consider the combined use of both approaches.

Figure 3 shows a comparison between the simulation of the use of risk neutral policy with $\ell_{t,n}$ limits and the combined $\ell_{t,n}$ and AV@R approach. It is clear that the frequency of low stored volumes is lower with the combined approach, for all the minimum levels considered.

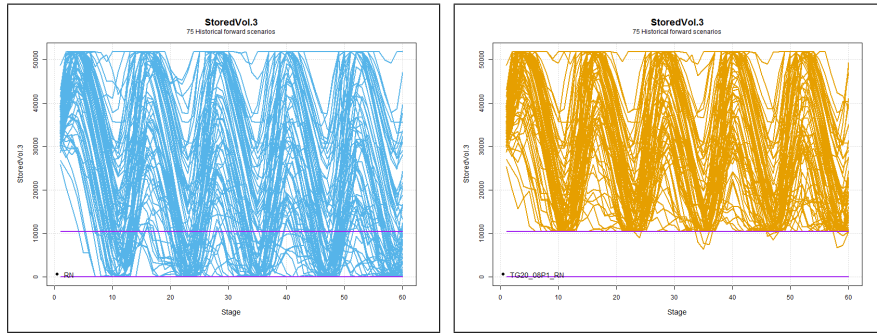
Figures 4, 5, 6 and 7 illustrate the performance of single $\ell_{t,n}$ and combined $\ell_{t,n} + \text{AV@R}$ approaches considering $\ell_{t,n} = 20\% \overline{\text{SE}}$ for all t, n and $\ell_{t,n} = \text{CAR}$. The effect of the CAR constraint is clearly different from the constant one, as can be seen in Figure 5 for the South subsystem.



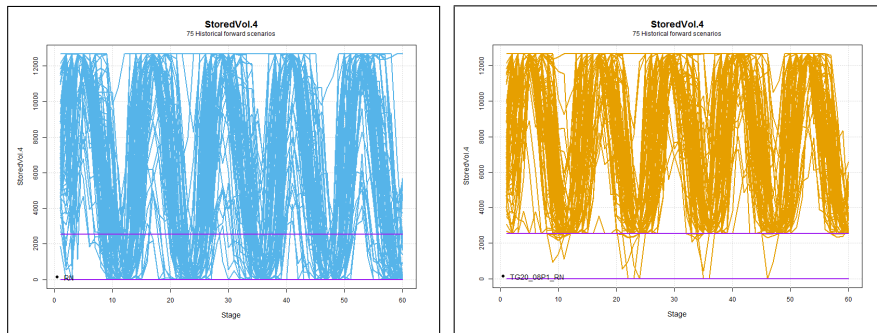
Southeast subsystem



South subsystem



Northeast subsystem

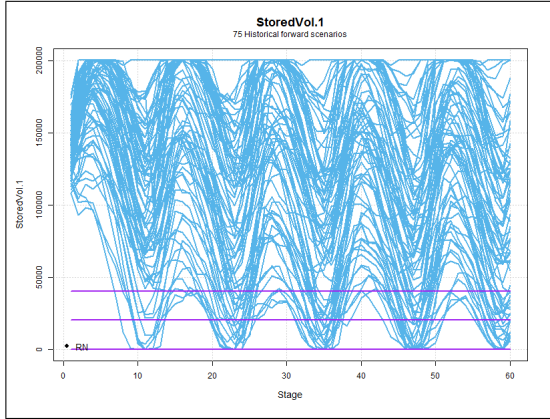


North subsystem

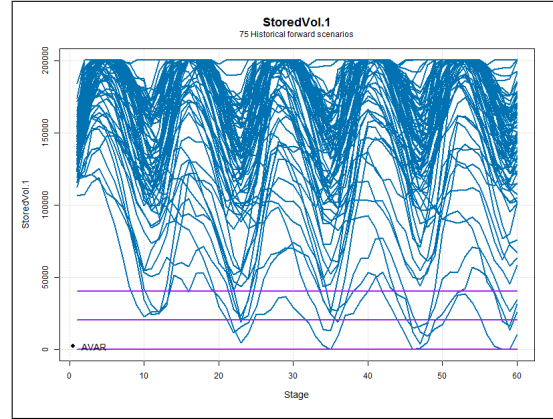
(a) Risk neutral; $\ell_{t,n} = 0$

(b) Risk neutral; $\ell_{t,n} = 20\% \overline{SE}$

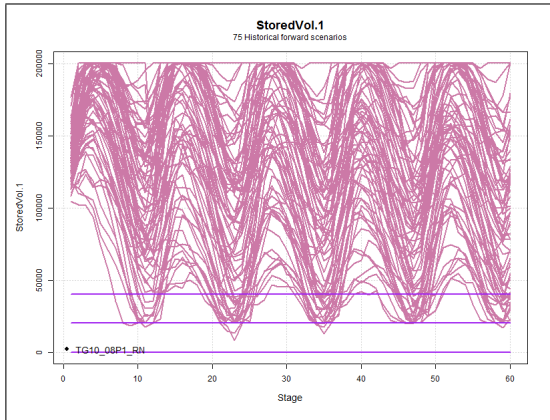
Figure 2: Risk neutral constant target level performance – Subsystems stored energy evolution (MWave).



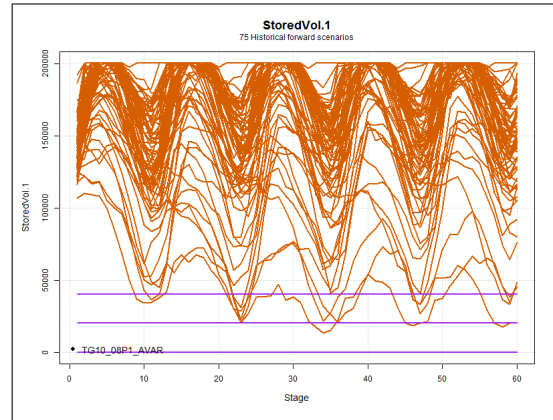
(a) Risk neutral; $l_{t,n} = 0$



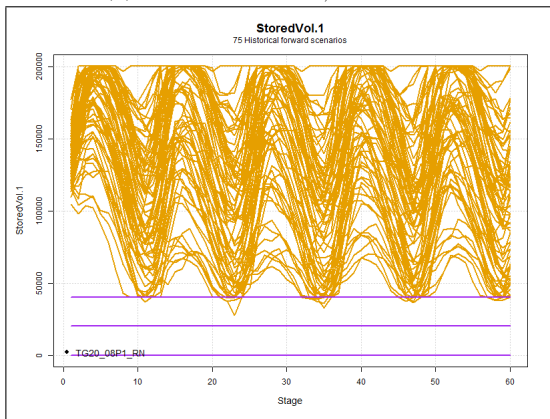
(b) AV@R; $l_{t,n} = 0$



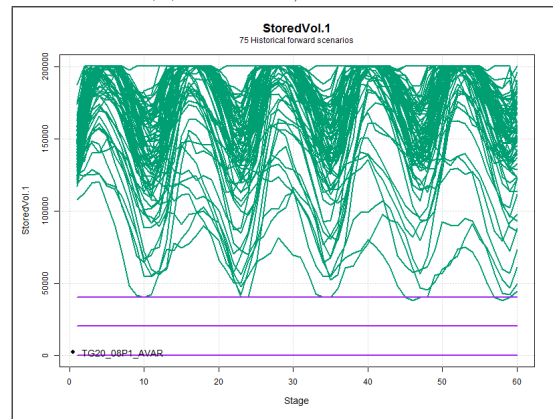
(c) Risk neutral; $l_{t,n} = 10\% \overline{SE}$



(d) AV@R; $l_{t,n} = 10\% \overline{SE}$



(e) Risk neutral; $l_{t,n} = 20\% \overline{SE}$



(f) AV@R; $l_{t,n} = 20\% \overline{SE}$

Figure 3: Southeast subsystem stored energy evolution (MWave) – Constant target level.

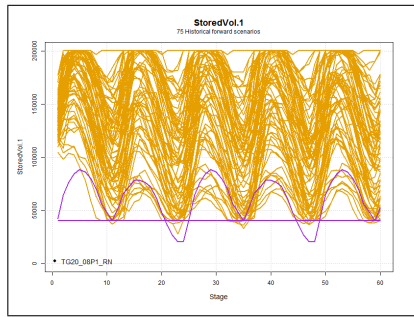
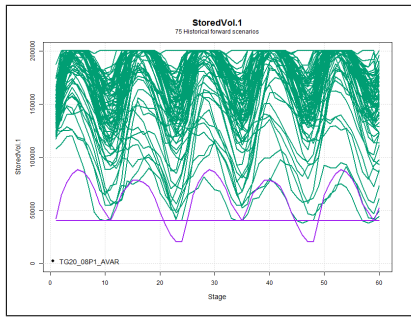
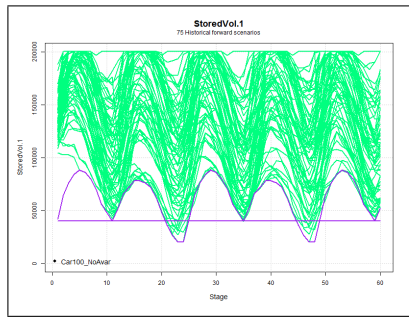
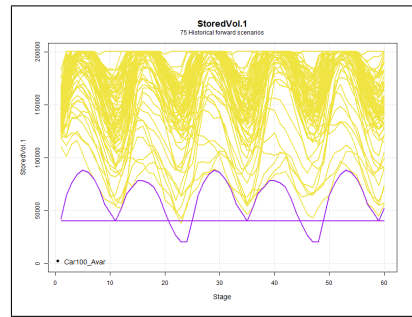
(a) Risk neutral; $\ell_{t,n} = 20\% \overline{SE}$ (b) AV@R; $\ell_{t,n} = 20\% \overline{SE}$ (c) Risk neutral; $\ell_{t,n} = \text{CAR}$ (d) AV@R; $\ell_{t,n} = \text{CAR}$

Figure 4: Stored energy for Southeast subsystem (MWave) – Constant and variable (CAR) target levels.

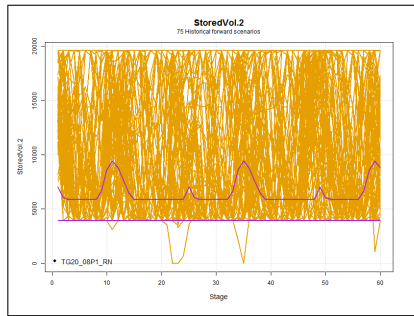
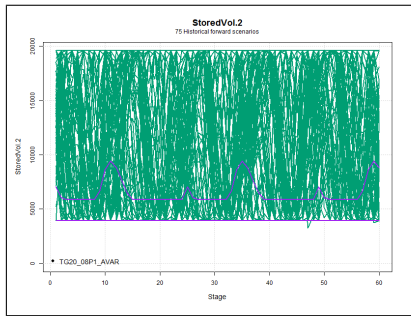
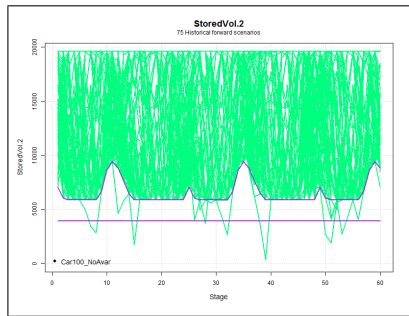
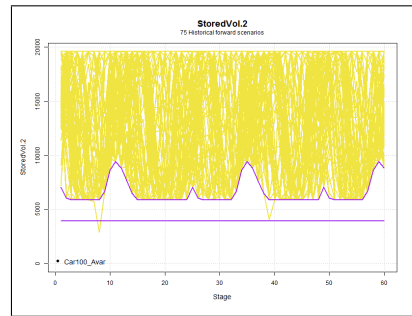
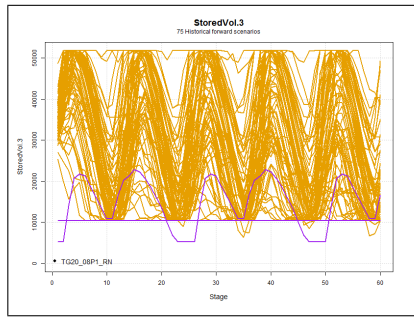
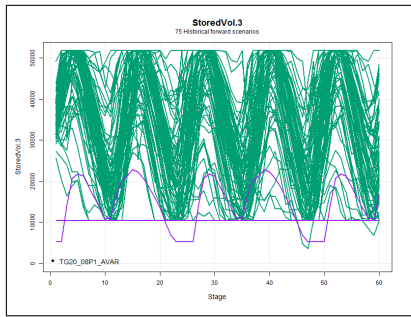
(a) Risk neutral; $\ell_{t,n} = 20\% \overline{SE}$ (b) AV@R; $\ell_{t,n} = 20\% \overline{SE}$ (c) Risk neutral; $\ell_{t,n} = \text{CAR}$ (d) AV@R; $\ell_{t,n} = \text{CAR}$

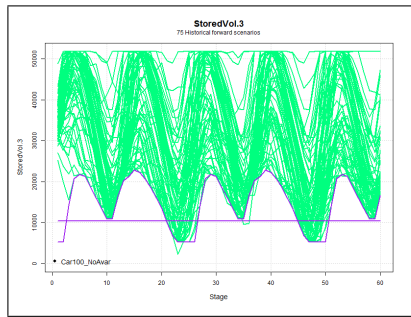
Figure 5: Stored energy for South subsystem (MWave) – Constant and variable (CAR) target levels.



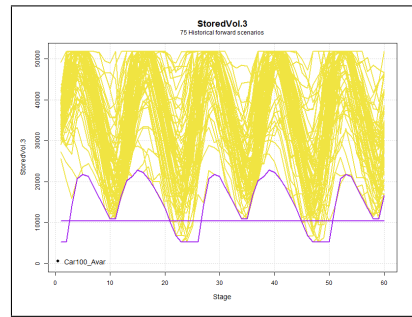
(a) Risk neutral; $l_{t,n} = 20\% \overline{SE}$



(b) AV@R; $l_{t,n} = 20\% \overline{SE}$

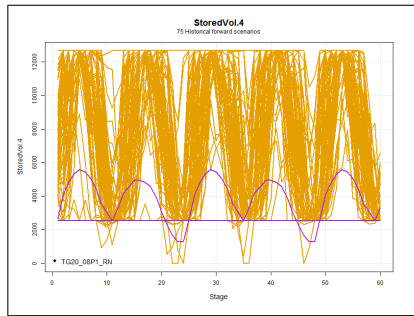


(c) Risk neutral; $l_{t,n} = \text{CAR}$

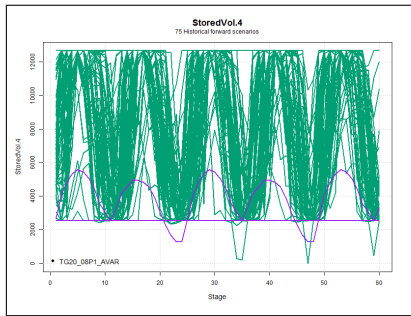


(d) AV@R; $l_{t,n} = \text{CAR}$

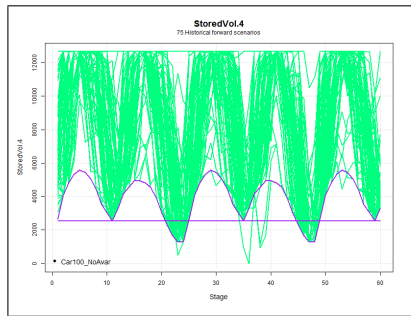
Figure 6: Stored energy for Northeast subsystem (MWave) – Constant and variable (CAR) target levels.



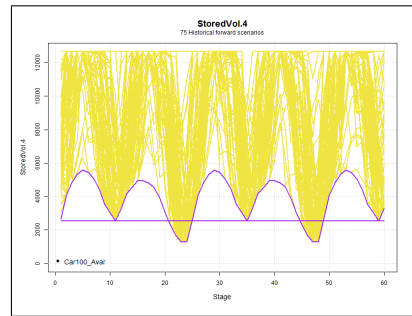
(a) Risk neutral; $l_{t,n} = 20\% \overline{SE}$



(b) AV@R; $l_{t,n} = 20\% \overline{SE}$



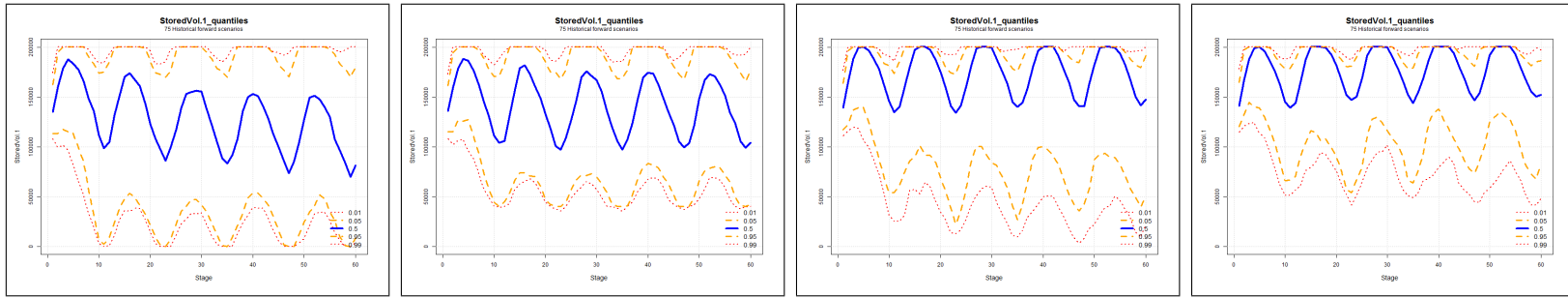
(c) Risk neutral; $l_{t,n} = \text{CAR}$



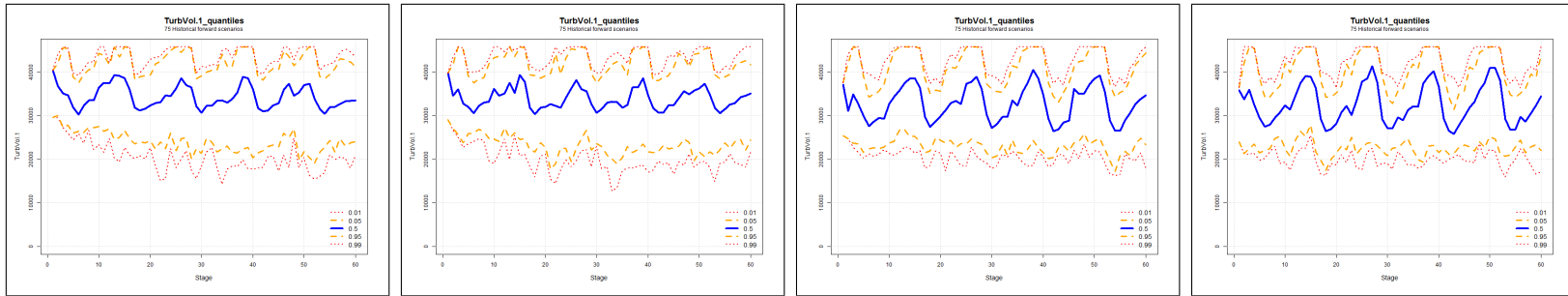
(d) AV@R; $l_{t,n} = \text{CAR}$

Figure 7: Stored energy for North subsystem (MWave) – Constant and variable (CAR) target levels.

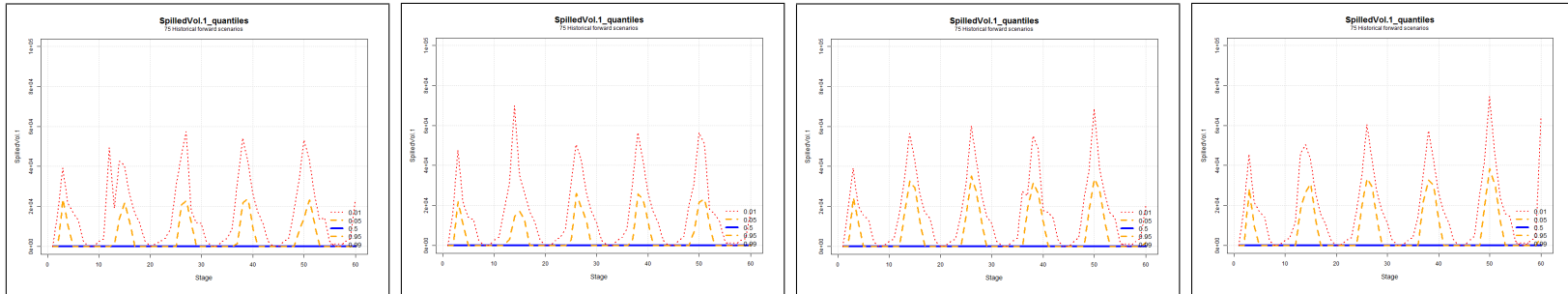
A comparison among these different policies effects on some variables is shown in Figures 8 and 9 and Table 3. As expected, the average stored energy, the spilled energy, the thermal generation and the expected value of the non-supplied energy (EENS) increase, while the hydro generation decreases as the risk averse concerns progressively increases from risk neutral to $\ell_{t,n} + \text{AV@R}$, although for thermal generation, spilled energy and EENS these changes are not very significant.



(a) Stored energy



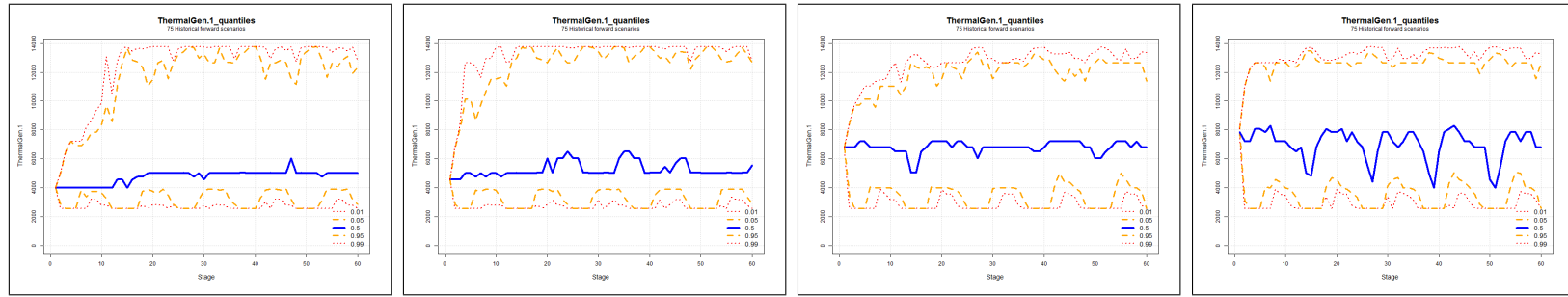
(b) Hydro generation



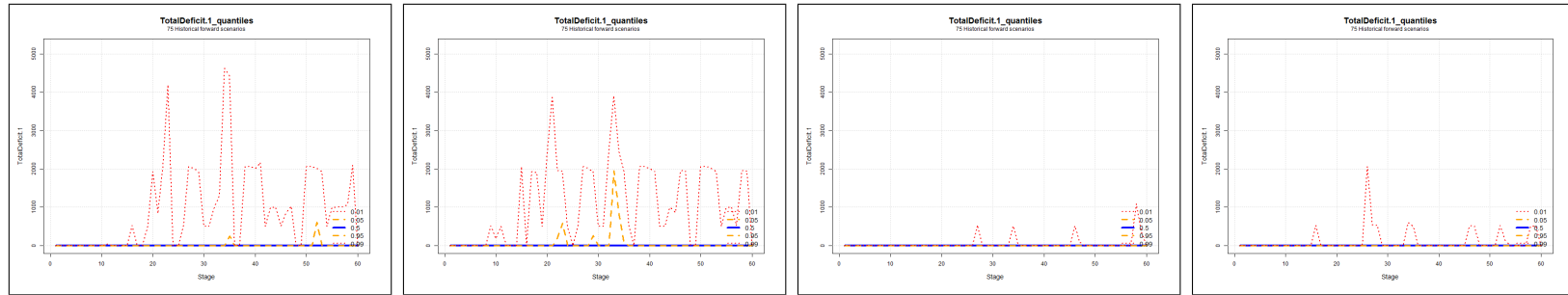
(c) Spilled energy

Risk neutral; $\ell_{t,n} = 0$ Risk neutral; $\ell_{t,n} = 20\% \overline{SE}$ AV@R; $\ell_{t,n} = 0$ AV@R; $\ell_{t,n} = 20\% \overline{SE}$

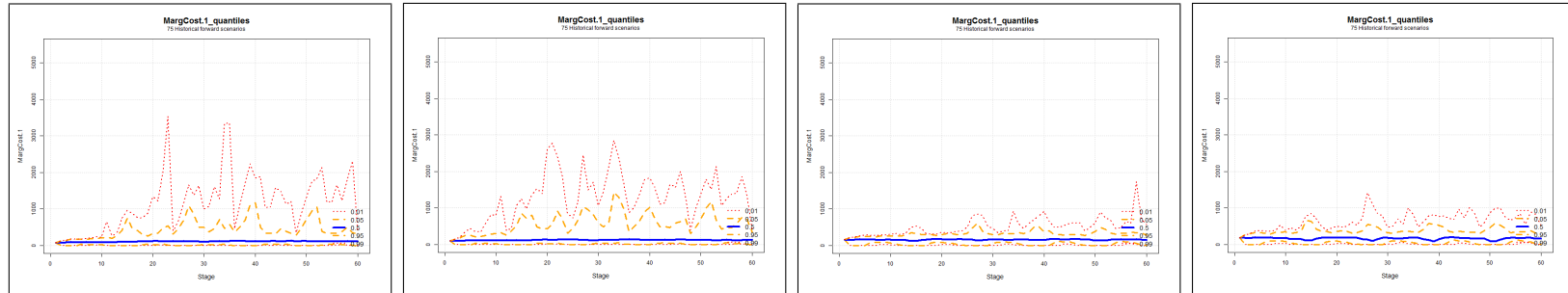
Figure 8: Sensitivity to risk averse criteria – Southeast variables quantile evolution (MWave).



(a) Thermal generation (MWave)



(b) Deficit energy (MWave)



(c) Marginal cost (\$/MWave)

Risk neutral; $\ell_{t,n} = 0$ Risk neutral; $\ell_{t,n} = 20\% \overline{SE}$ AV@R; $\ell_{t,n} = 0$ AV@R; $\ell_{t,n} = 20\% \overline{SE}$

Figure 9: Sensitivity to risk averse criteria – Southeast variables quantile evolution.

Case study	Stored	Hydro	Spilled	Thermal	EENS
RN	160,705.96	55,375.18	3002.01	7,968.74	60.07
RN 10%	170,435.84	55,013.53	3097.72	8,334.73	55.73
RN 20%	183,077.21	54,597.93	3224.25	8,739.45	66.61
RN CAR	187,788.50	54,409.07	3303.53	8,909.47	85.46
AV@R	206,889.12	53,479.81	3753.47	9,919.57	4.62
AV@R 10%	213,463.86	53,176.44	3914.77	10,219.37	8.19
AV@R 20%	221,781.99	52,818.10	4138.79	10,576.99	8.91
AV@R CAR	225,256.88	52,630.63	4233.67	10,760.63	12.74

Table 3: NIPS mean total energy sensitivity to risk averse criteria (MWave).

An analysis of the cost impact of imposing the $\ell_{t,n}$ limits criterion is shown in Figure 10, where the impact of such measure can be easily noticed. This can also be seen in Table 4 which contains the average total operation cost, comprised by thermal and deficit costs, total penalty values and total deficit costs averaged over the 75 historical sequences.

Note that, as the target level penalty increases, in order to keep the stored volumes above the minimum target it becomes cheaper to curtail the load and pay the corresponding deficit cost.

It is instructive to notice in Figure 11 that the marginal costs variability of the risk neutral approach is reduced with AV@R methodology and imposing the $\ell_{t,n}$ targets does not change this behavior.

Case study	Operation cost	Penalty cost	Deficit cost
RN	57,163.146	0.000	8.605
RN 10%	61,567.589	2.114	6.078
RN 20%	68,452.488	2.621	7.712
RN CAR	73,555.497	2.814	11.975
AV@R	68,662.333	0.000	0.542
AV@R 10%	74,240.009	0.786	0.838
AV@R 20%	80,360.223	0.559	0.940
AV@R CAR	83,603.501	0.178	1.468

Table 4: Stored energy target level sensitivity – Mean total costs (10^6 \$).

In summary, the above results show that:

- the target levels $\ell_{t,n}$ prevent that the energy reservoir depletes below the prescribed values for most of the historical simulated sequences;
- $\ell_{t,n}$ and AV@R approaches work well together, with the additional benefit of frequency reduction of low levels along time.
- the mean cost impact of imposing $\ell_{t,n} = 10\% \overline{SE}$ is around 10% of the total operation cost for both risk neutral and AV@R criteria.

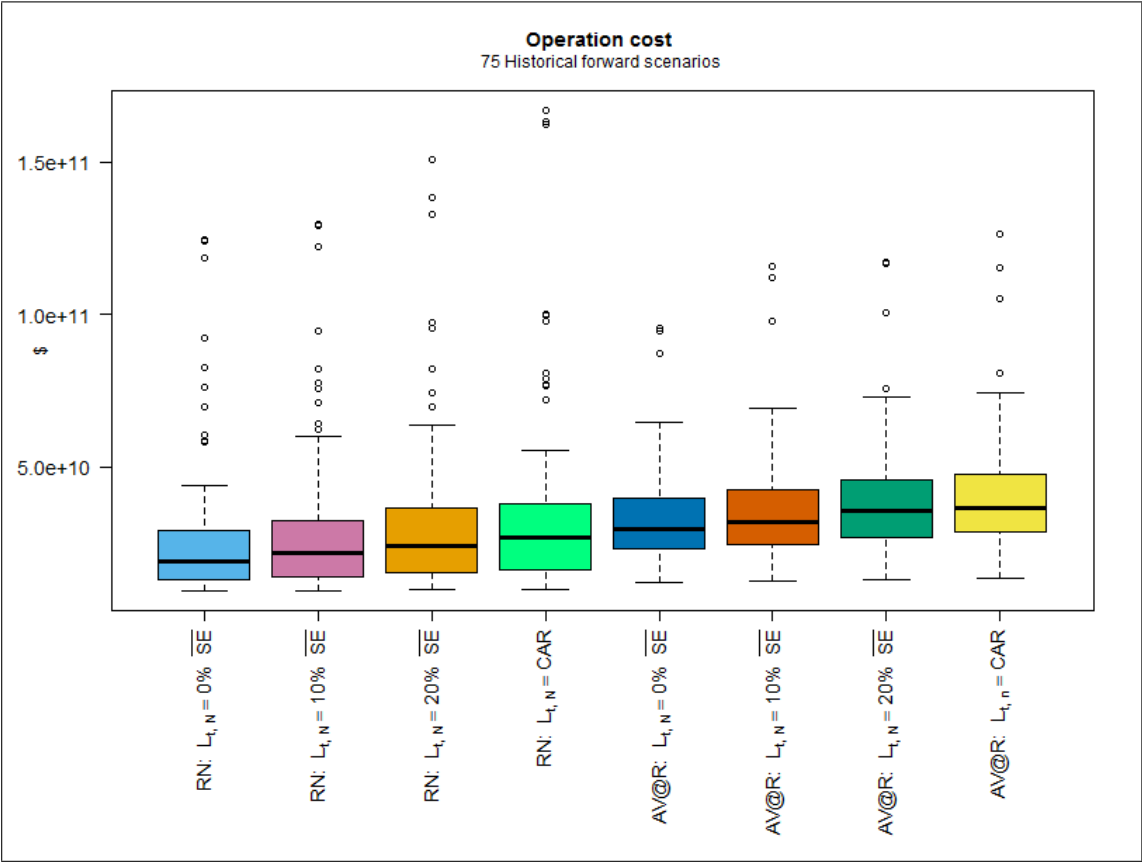
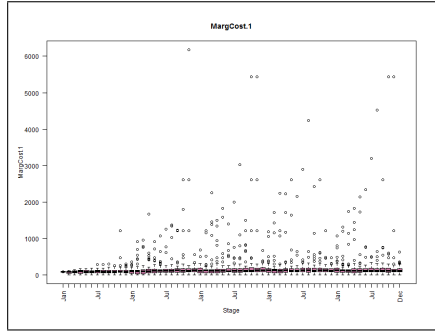
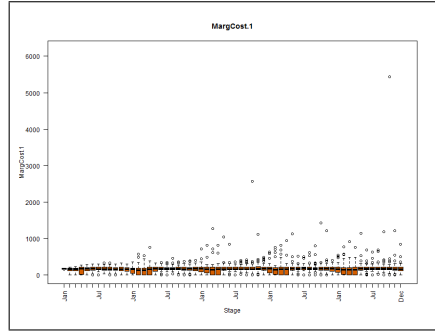


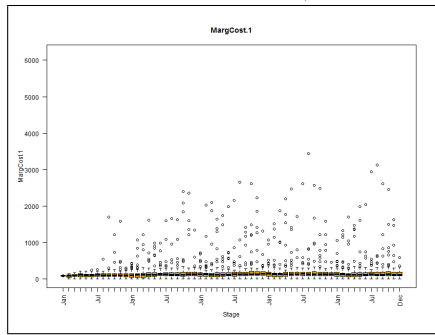
Figure 10: Total Operation Costs (10⁶ \$).



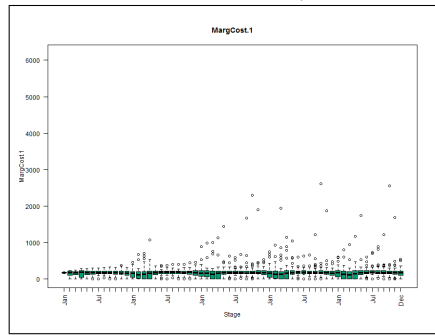
(a) Risk neutral: $\ell_{t,n} = 0$



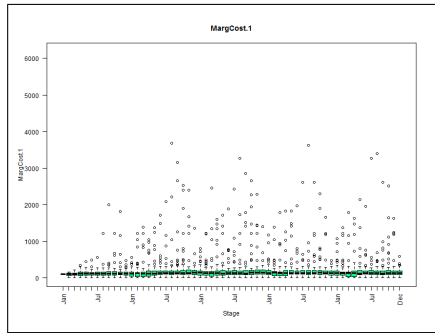
(b) Risk averse: $\ell_{t,n} = 0$



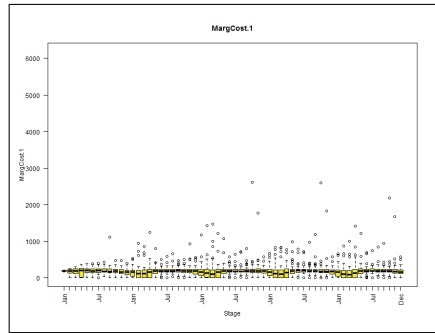
(c) Risk neutral: $\ell_{t,n} = 10\% \overline{SE}$



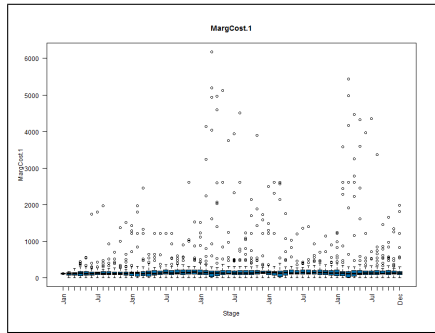
(d) Risk averse: $\ell_{t,n} = 10\% \overline{SE}$



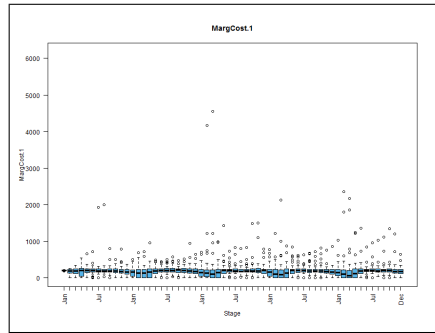
(e) Risk neutral: $\ell_{t,n} = 20\% \overline{SE}$



(f) Risk averse: $\ell_{t,n} = 20\% \overline{SE}$



(g) Risk neutral: $\ell_{t,n} = \text{CAR}$



(h) Risk averse: $\ell_{t,n} = \text{CAR}$

Figure 11: Southeast marginal costs sensitivity considering different minimum target levels (\$/MWave).

3.2 Choosing penalty

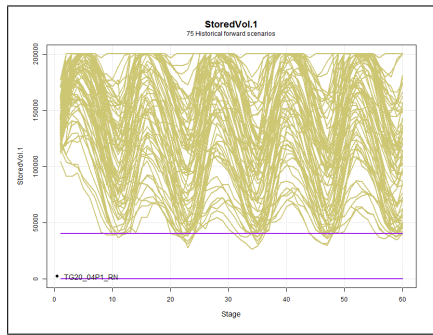
In order to estimate the impact of different penalty values, a study considering 0.4, 0.6, 0.8 and 0.9 of the first depth deficit cost (P1) was done. In this analysis, the subsystem North was included to emphasize the effect of changing the amount of penalty because, among all four subsystems, it has the highest number of violations of minimum target level.

Figure 12 shows that imposing the $\ell_{t,n}$ limits results in a noticeable effect in preventing the depletion of the stored energy in the reservoirs, even for values as small as 0.4P1 for the penalty.

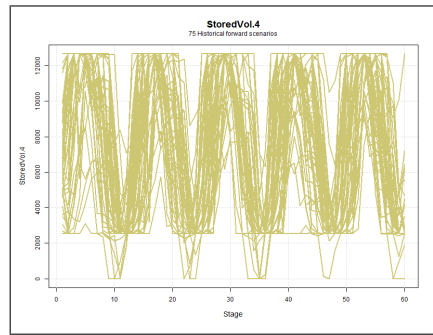
The results on Table 5 show the same pattern of Table 3. One can see that with the use the combined $\ell_{t,n}$ and AV@R approach and penalty as small as 0.4P1, there is almost no increase of the EENS.

Table 6 shows that, for the $\ell_{t,n}$ risk neutral approach, as the minimum stored energy target penalty increases, the total deficit cost also increases, meaning that is more profitable to maintain the reservoirs filled than to meet the demand. It is noteworthy see that, if one chooses to use the combined $\ell_{t,n}$ and AV@R approach, the penalty and deficit costs are much smaller, although the total cost operation is higher due to the more intense use of thermal generation.

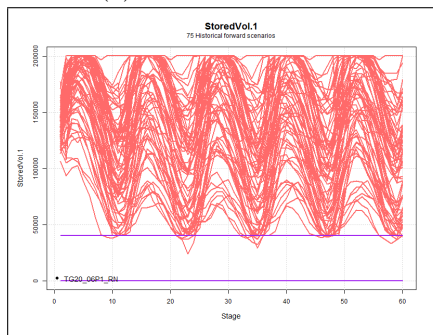
Remark 2 *The definition of a most proper penalty value is not the object of this report and must be performed in future work.*



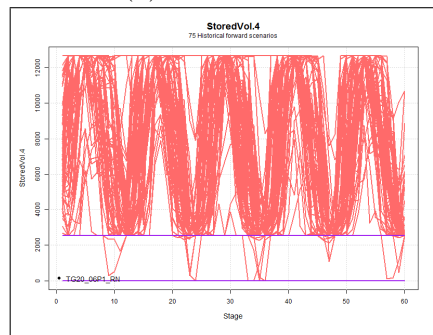
(a) Southeast – 0.4P1



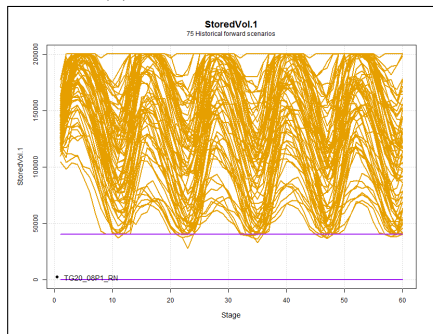
(b) North – 0.4P1



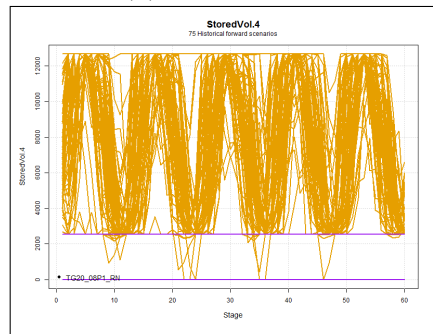
(c) Southeast – 0.6P1



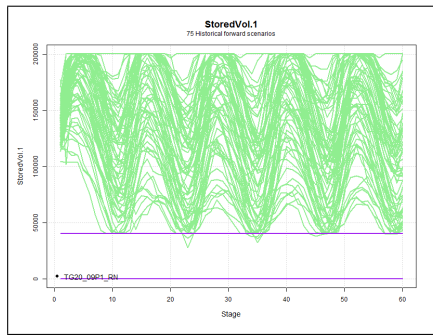
(d) North – 0.6P1



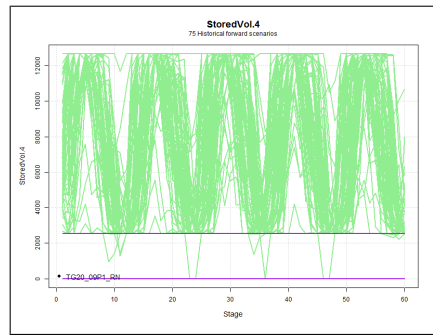
(e) Southeast – 0.8P1



(f) North – 0.8P1



(g) Southeast – 0.9P1



(h) North – 0.9P1

Figure 12: Sensitivity to penalty values – Subsystem stored energy evolution (MWave).

Case study	Stored	Hydro	Spilled	Thermal	EENS
RN	160,705.96	55,375.18	3,002.01	7,968.74	60.07
RN 0.4P1	181,391.21	54,638.14	3,221.80	8,719.64	46.22
RN 0.6P1	182,474.69	54,610.57	3,226.81	8,734.05	59.37
RN 0.8P1	183,077.21	54,597.93	3,224.25	8,739.45	66.61
RN 0.9P1	183,208.28	54,584.65	3,234.49	8,747.55	71.79
RN 1.0P1	183,098.32	54,566.59	3,241.87	8,766.02	71.39
AV@R	206,889.12	53,479.81	3,753.47	9,919.57	4.62
AV@R 0.4P1	220,851.69	52,846.55	4,113.85	10,552.66	4.79
AV@R 0.6P1	221,617.13	52,802.99	4,149.09	10,593.67	7.34
AV@R 0.8P1	221,781.99	52,818.10	4,138.79	10,576.99	8.91
AV@R 0.9P1	222,221.43	52,778.49	4,169.74	10,615.30	10.21
AV@R 1.0P1	222,723.16	52,774.56	4,152.97	10,619.59	9.85

Table 5: Penalty sensitivity – NIPS mean total energy (MWave).

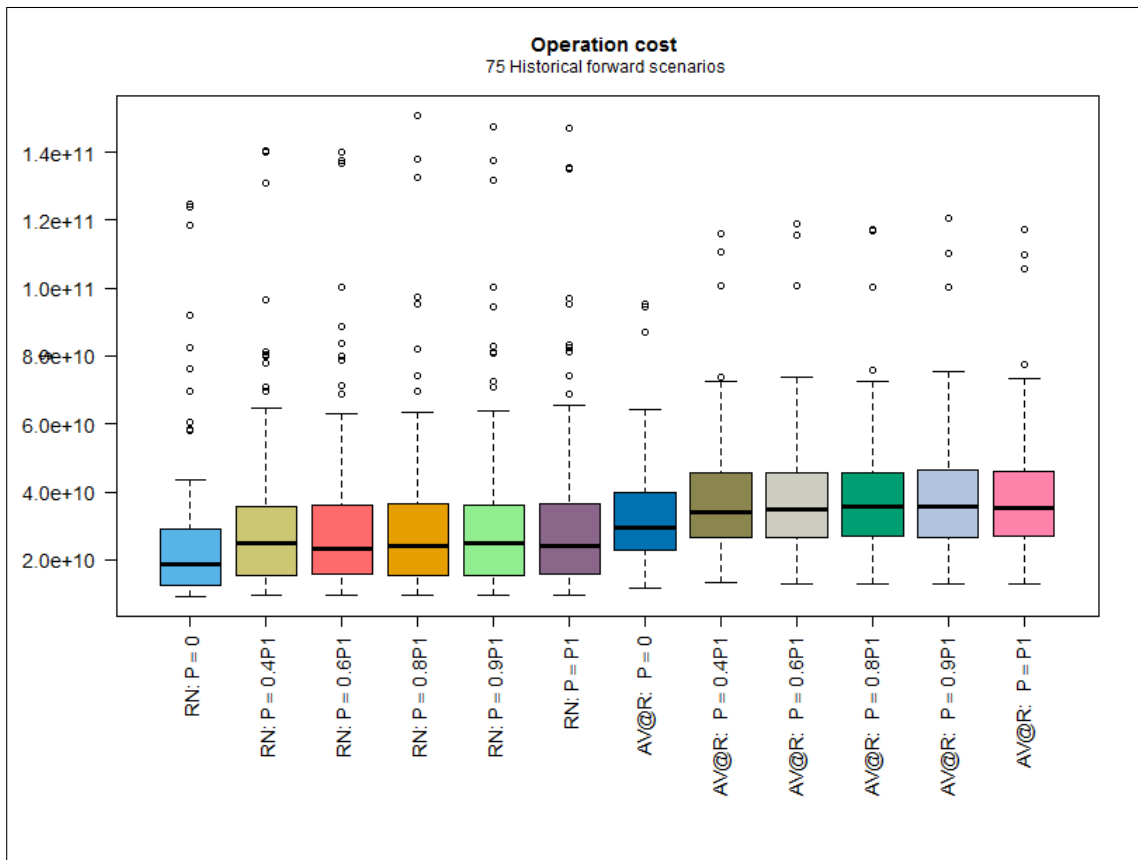


Figure 13: Total Operation Costs

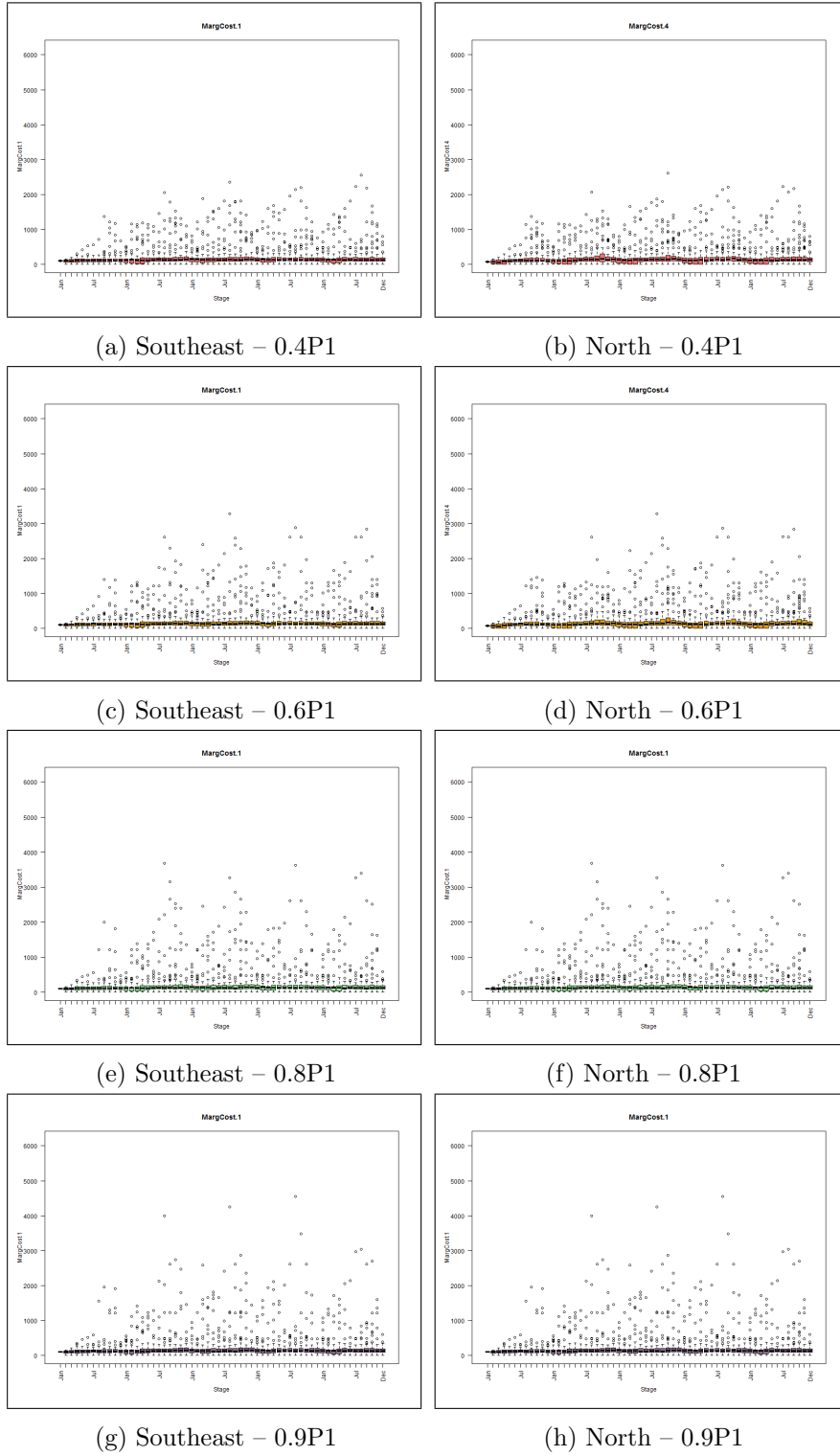


Figure 14: Sensitivity to penalty values, $\ell_{t,n}$ approach – SE and N subsystem marginal costs evolution (\$/MWave).

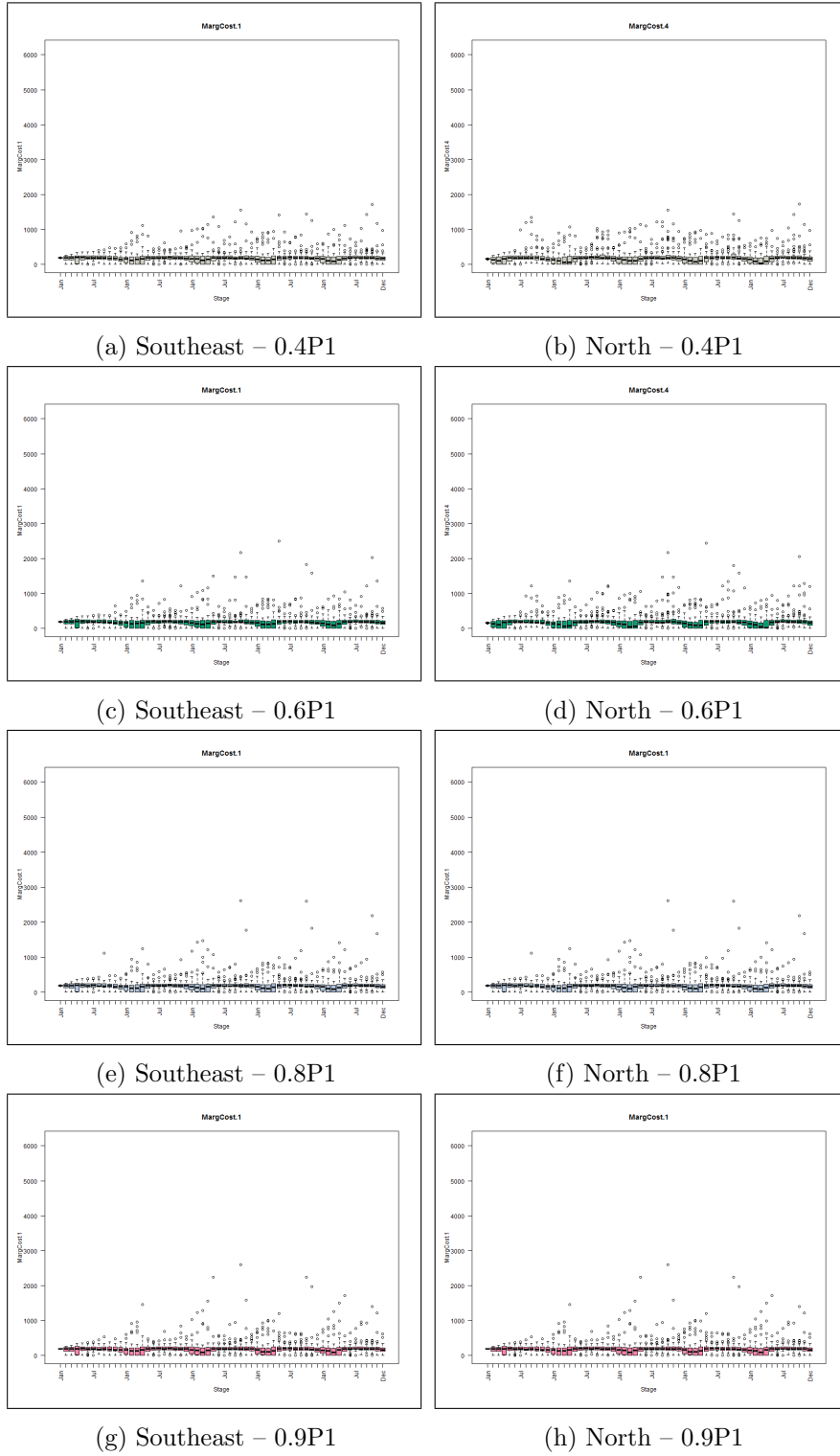


Figure 15: Sensitivity to penalty values, $\ell_{t,n}$ and AV@R combined approach – SE and N subsystem marginal costs evolution (\$/MWave).

Case study	Operation cost	Penalty cost	Deficit cost
RN	57,163.147	0.000	8.605
RN 0.4P1	67,304.456	4.020	4.760
RN 0.6P1	67,896.778	2.805	6.597
RN 0.8P1	68,452.489	2.621	7.713
RN 0.9P1	68,507.863	2.050	8.339
RN 1.0P1	68,624.342	1.574	8.521
AV@R	68,662.333	0.000	0.542
AV@R 0.4P1	79,688.369	0.000	0.467
AV@R 0.6P1	80,586.105	0.000	0.747
AV@R 0.8P1	80,360.224	0.559	0.940
AV@R 0.9P1	80,792.804	0.000	1.055
AV@R 1.0P1	81,193.594	0.302	1.156

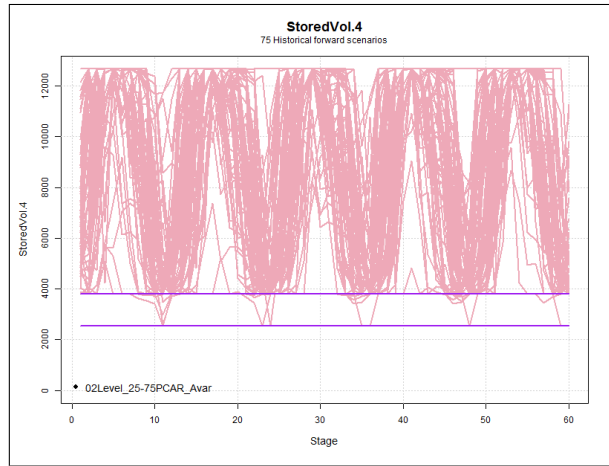
Table 6: Penalty sensitivity – Mean total costs (10^6 \$).

3.3 Multilevel minimum target curves

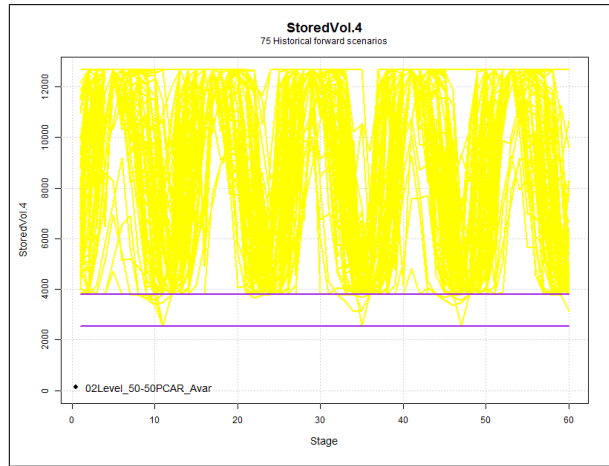
An additional approach which considers more than a single target curve combined with the AV@R was carried out.

One study considered two constant target levels ($\ell_{t,n} = 30\% \overline{SE}$ and $\ell_{t,n} = 20\% \overline{SE}$) with associated penalties given by $((.50, .50), (.75, .25)$ and $(.25, .75)) * 1130\$/MWave$. The performance is illustrated in Figure 16 for the North subsystem, where one can note the effect of both active stored energy target levels. For instance, in case (a) there is a greater number of series in the range 30-20% whereas in case (c) this number is lower. Most of the series remain above the highest target level and the depletion of the reservoir of those below stops above the lower target level.

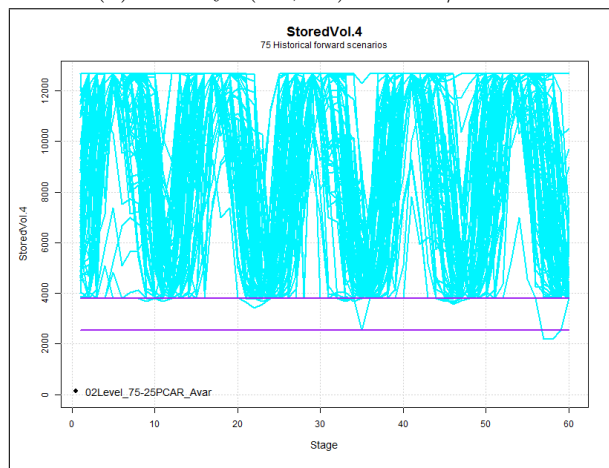
The second study considered multiple variable target curves: CAR, CAR & 1.1*CAR and CAR & 1.1*CAR & 1.2*CAR. The total penalty (1130 \$/MWave) is equally divided among the number of active curves. The same pattern can be noticed in both Figures 17 and 16.



(a) Penalty: $(.25, .75) \times 1130\$/MWave$

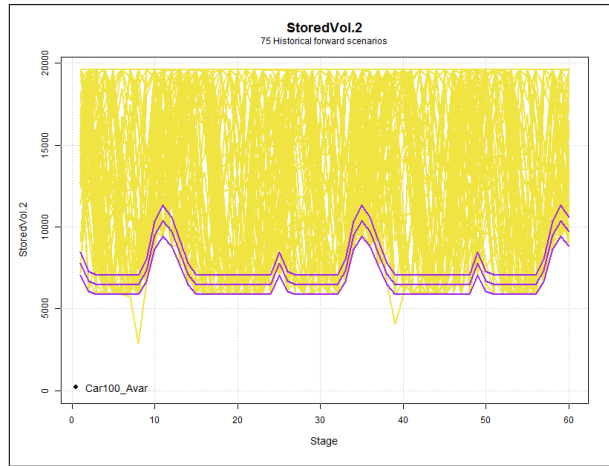


(b) Penalty: $(.50, .50) \times 1130\$/MWave$

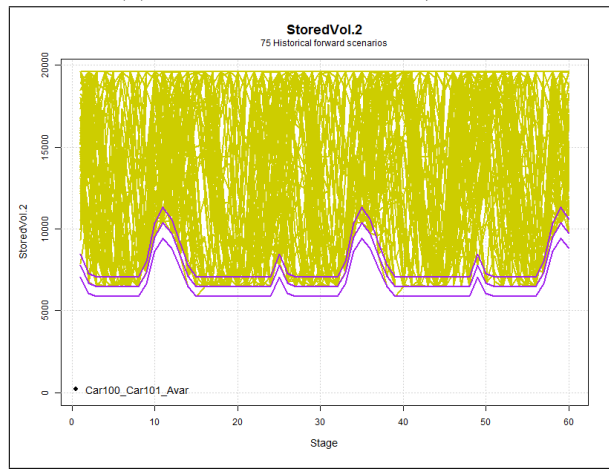


(c) Penalty: $(.75, .25) \times 1130\$/MWave$

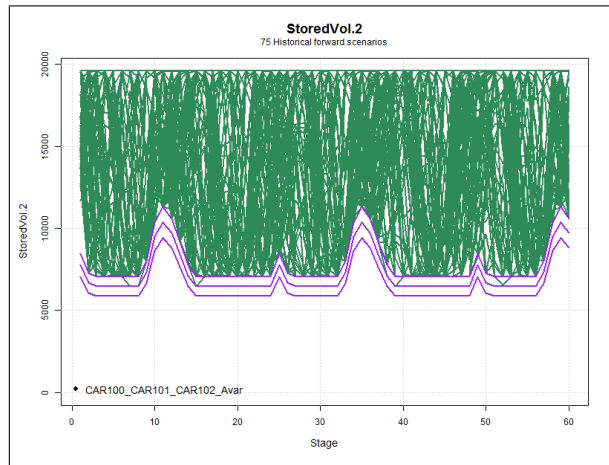
Figure 16: Multiple constant target level $(30\%, 20\%) \times \overline{SE}$ and AV@R performance for North system (MWave)



(a) CAR, penalty = 1130\$/MWave



(b) CAR & 1.10 CAR, penalty = 565 & 565\$/MWave



(c) CAR & 1.10 CAR & 1.20 CAR, penalty = 377 & 377 & 377 \$/MWave

Figure 17: Multiple variable target level CAR & 1.10 CAR & 1.20 CAR performance for South system (MWave)

4 Conclusion

In this report, in order to limit the depletion of the equivalent energy reservoir, the use of a minimum stored energy target curve is investigated. This is achieved by imposing a penalty every time the stored volume goes below the target level. This target curve can be taken as constant along time, as well as a variable one, like a user provided Risk Aversion Curve (CAR). Moreover, multiple energy target curves can be considered in order to represent the increasing concern as the energy storage level becomes low, due to increasing operation difficulties.

Even if imposing these limits fulfils the intended purpose, with the sole use of the limit curves the stored volume frequently touches the limit curve and the associated operation costs tend to have high variability.

On the other hand, it is known that the AV@R risk averse criterion limits the occurrence of high operation costs but does not prevent the emptying of the reservoir.

This report describes minimum stored energy target curve methodology and provides methodological support for the combined use of both approaches to control the evolution of the stored energy along the stages of the operation period. The numerical experiments conducted considering the combined approach support the adequacy of its use.

The main take-aways are:

- The inclusion of $\ell_{t,n}$ constraint results in:
 - reduced occurrence of stored energy values below the targets;
 - increase in thermal generation;
 - increase the expected value on not supplied energy (EENS) with levels and/or penalty.
- Combined $\ell_{t,n}$ + AV@R approach results in:
 - less cost variability (volatility) when compared with the risk neutral CAR approach;
 - higher stored energy levels when compared to the AV@R approach;
 - reduced frequency of low stored energy levels when compared to the minimum stored energy target level (risk neutral);
 - the total penalty cost is smaller than the ones associated with the risk neutral.

Remark 3 *The main objective of this report is to evaluate the proposed methodology and not to define the most proper target levels or penalty values. This investigation can be performed in future work.*

Remark 4 *Most case-studies were performed using 0.8P1 as penalty.*